KINETO-ELASTODYNAMIC ANALYSIS OF SLIDER-CRANK MECHANISM WITH FLEXIBLY ATTACHED SLIDER

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TO MY PARENTS

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AVADHESH KUMAR KHARE

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KINETO - ELASTODYNAMIC ANALYSIS OF SLIDER - CRANK MECHANISM WITH FLEXIBLY ATTACHED SLIDER

The mechanism examined is the common slider - crank mechanism in which the slider mass is connected with the coupler by means of a linear spring. The crank and the coupler are elastic and their masses and inertias have also been considered. Equations for Kineto-Elastodynamic Analysis of the Mechanism have been derived and an iterative scheme is suggested to solve them. Friction is assumed small and the effect of clearances in joints and tolerances on links have been neglected. Some example problems are worked out and the results are compared with the available ones.

CHAPTER I

INTRODUCTION

1.1 General Introduction

During last fifteen years a lot of work has been done in the field of design of mechanisms. Many investigators have contributed towards kinematic synthesis of mechanisms and it has proved to be a powerful design tool for engineers. Almost all the techniques of kinematic synthesis of mechanisms, work well and give acceptable results for low-speed mechanism. However, in common, they suffer with one major shortcoming. This is the "rigidity" assumption which prevails throughout the literature with few notable exceptions.

In high - speed mechanisms the inertia forces are large enough to cause speed - fluctuations as well as appreciable elastic deformations in the links of the mechanism, which can not be overlooked. The effect of inherent elasticity of a mechanism system may not be pronounced in low speed light weight mechanisms, but in high speed mechanisms and in particular in high precision mechanisms, the analysis may not be perfect unless this effect is considered. For example a kinematically synthesized gripper mechanism considering all of its links rigid, may miss the target if it is run at a speed more than one - half of its design speed.

In certain machines with reciprocating or oscillating motions proper timing of the motion is a prerequisite to smooth

operations. At high speeds owing to the large inertia forces, the main drive undergoes fluctuation of speed and the link - members of the mechanism are elastically deformed. This causes a difference between the actual kinematic characteristics of the mechanism at certain predetermined positions of the main drive and that predicted from the point of view of rigid body assumptions.

The deviation of the mechanism characteristics from the expected ones perhaps motivated the workers to search for some new analysis and synthesis techniques in which the effect of inherent elasticity of mechanism system was taken into account. The analysis procedures of the mechanisms have been grouped in the following main categories. [11]

Static Analysis : Determination of Inertia forces, stresses, strains and deflections in members and joints due to external and / or gravitational loading.

Kinematic Analysis : Examination of the displacement, velocity ratios, acceleration ratios etc., of a mechanism with all of its members regarded as rigid. The reference variable is usually a position parameter.

Dynamic Analysis : Determination of the displacements, velocities accelerations etc. of a mechanism, including derivations of inertia forces of a mechanism made up of rigid members. The reference variable is time.

Elastic Analysis : Examination of stresses and deflections of an elastic system due to static load in order to determine system flexibilities or stiffness.

Elastodynamic Analysis : Examination of displacements, velocities, accelerations, stresses, strains etc., of a moving elastic mechanism. Inertia forces are calculated by assuming all the members rigid.

Kineto-elastodynamic Analysis (KEDA) : Examination of the displacements, velocities, accelerations, stresses, strains etc., of a moving elastic mechanism. Effects of elastic deformation upon the inertia forces are included in the analysis.

An ideal analysis procedure of high speed mechanisms should not only consider the fluctuations in speed of the main drive owing to the inertia forces and the inherent elasticity of the mechanism, but it should also take into account the damping characteristics of the system, clearances in the joints, tolerances on the links of the mechanism etc. However, such an analysis will be very much complicated and nobody hitherto has analysed the mechanisms considering all these effects together. However, attempts have been made to dynamically analyse and synthesize mechanism with assumptions of varied degrees. In some cases particular links were considered to be elastic and masses to be concentrated at the hinges and so on. The analysis and synthesis of slider - crank mechanism seemed to have, judging from the published papers on this topic, attracted least attention of all the planar mechanisms.

1.2 Literature Survey

During past few years valuable contributions have been made in the field of dynamics of mechanisms. Perhaps, due to the inaccurate response of the kinematically synthesized mechanisms at high speeds, the literature has recognized the need for dynamic analysis and synthesis techniques. However, in a great majority of work the dynamic analysis has been performed considering rigid links.

Early work in dynamics of mechanisms concerned themselves with deriving the velocities, accelerations etc. of mechanisms. Quinn [31, 32] has pioneered in the energy distribution method and has set the stage for further work. For determining dynamic characteristics of mechanisms, Hirschhorn [16] has given the "rate of change of energy approach" in which the time rate of change of total energy of the mechanical system is equated to the power input. Thus, if the power input function is known, this method will yield the time response of the system.

Elastic deflections of significant magnitude in high speed mechanisms due to their inherent elasticity have attracted the workers and various techniques have been developed to account for this effect. The use of springs in the mechanisms have also been made in order to alter the dynamic performance of the given mechanism in high speed motion. This may improve the response of the system, reduce the shaking forces transmitted to the frame or support or produce a performance which would otherwise be unsatisfactory with

the rigid member mechanism. Van Sickle and Goodman [43] used springs to adjust the speed of mechanisms. Imam [19] examined the response of a four - bar function generator with a longitudinal spring as a part of its coupler. Davidson [5] has examined a slider - crank mechanism with the slider mass connected by a spring. In [40] Benedict states that "the addition of springs is one of the easiest ways to balance torque and obtain a smooth - running machine".

Some authors [2, 3, 19, 20, 25] have dealt with "elastic complex" systems i.e. a mixed mechanism in which few members are elastic and the remaining are taken to be rigid. Lagrangian mechanics or Energy methods are used to derive the equation of motion. Generally, as the solution of these problems is tedious, only one member is taken to be elastic and that too with only one elastic degree of freedom.

Dubowsky and Freudenstein [6] have made notable contribution to the study of backlash and have modeled an elastic mechanical joint, which they have called as "impact pair", and have determined the dynamic equations for the relative motion of such joints.

Slider - crank mechanism has been the centre of study of several workers. Jasinski, Lee and Sandor [20, 21] have studied the stability of this mechanism having a flexible connecting rod. Neubauer, Cohen and Hall [29] have made a notable contribution by examining the vibrations of the elastic connecting rod of the slider - crank mechanism.

Liniecki [26] considers the fluctuation of input crank speed due to the effect of inertia forces. He deals with a procedure for the kinematic redesign of the slider - crank mechanism so as to minimise the error between the actual and the desired slider velocities at three predetermined positions of the crank.

Davidson [5] has examined a slider - crank mechanism connected in series with a spring and mass. He has taken crank and coupler to be massless and rigid and the only inertia load is due to the mass of the slider. He has also assumed a constant driving speed and with these assumptions he has synthesized the mechanism as a function generator.

Sheerwood [35] considers the dynamic synthesis of a slider-crank mechanism so as to minimise the error between the actual and the desired time taken by the slider to travel a prespecified distance. Sheerwood and Hockey [34, 37] have considered the redistribution of coupler mass of a slider - crank mechanism from balancing point of view.

The literature reviewed above has not provided a method for "Kineto - Elastodynamic Analysis" (KEDA) of mechanisms in which all effects are considered. Only recently some contributions have been made towards KEDA of mechanisms.

Erdman and Sandor [7] have called KEDA to be "a frontier" in mechanism design. Sadler and Sandor [33] used harmonic analysis

to analyse the kineto - elastodynamic deflections of the extension of the coupler of a four - bar path generator linkage. Erdman, Imam, Sandor and Oakberg [8, 9, 10, 19] have dealt with a general method for KEDA and Kineto - Elastodynamic Synthesis (KEDS) of mechanisms using Kineto - Elastodynamic Stretch Rotation Operator (KEDSRO). They have started with the kinematic analysis to get system forces and elastic deformations, derive input link acceleration using law of conservation of energy and get velocities and accelerations of remaining links from what they call "Kineto - Elastodynamic family tree", obtained by differentiation of KEDSRO. The entire procedure is iterated till a suitable convergence criterion is satisfied. Imam [18] deals with a KEDA and KEDS technique and without using "KEDSRO" synthesizes a mechanism with minimum weight criterion.

1.3 Aim Of The Present Work

As stated earlier kineto - elastodynamic analysis of high speed mechanisms is a recently developed field of study and deserves attention of mechanism designers. Particularly in high speed high precision mechanisms (e.g. gripper mechanisms of computers and printing machines etc.) the elastic deformations are large enough to affect the mechanism response and hence the desired accuracy can only be achieved if the effect of the inherent elasticity of the mechanism is also considered during analysis.

In the present work kineto - elastodynamic analysis of the eccentric slider - crank mechanism with flexibly connected slider has been performed. All the links have been considered to be elastic. The masses and inertia effects of all the links have also been considered. However, the effect of the clearances and tolerances, damping characteristics of the mechanism and the friction effects at the junction of two links have been neglected.

The common slider - crank mechanism is a particular case of the mechanism considered here when the stiffness of the spring connecting the slider is considered to be very high (theoretically infinite).

CHAPTER II

FORMULATION OF THE PROBLEM AND SOLUTION TECHNIQUE

2.1 Formulation Of The Problem

Figure 1 represents the mechanism under study. The stiffness k of the spring connecting the slider with the coupler is assumed to be linear. The crank and coupler are assumed to be linearly elastic and their elastic and section modulii are taken to be uniform and constant. The mass of the coupler is concentrated at its centre of gravity G (which is taken at its mid - way for simplicity.) The gravitational forces, being much small in comparision to inertia forces, are neglected. The joints are assumed to be frictionless and slider - friction has also not been accounted. There is no play in joints. Linear superposition of rigid - body and elastic deformations is assumed.

The mechanism driver rotates at an average angular velocity Ω. The main drive undergoes fluctuations of its speed due to inertia of crank, coupler and the slider mass and at the steady - state the input crank angular velocity $\dot{\theta}$ is a function of input angle θ only. The distance X_c represents the actual location of coupler end point C while X represents location of point D. When the mechanism is motionless point D on the slider coincides with point C on the coupler. For very large values of k (a rigid connection between slider and coupler) C and D would coincide for all

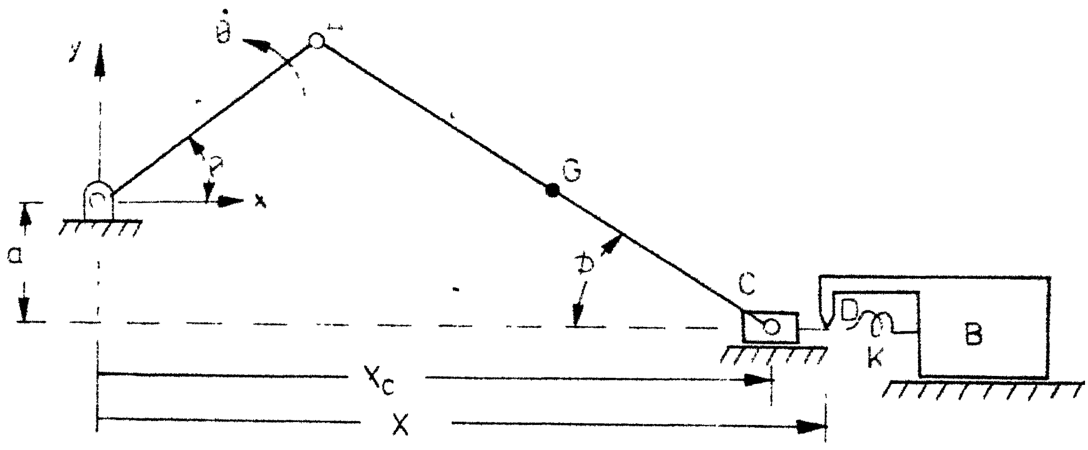

Fig. 1 Figure showing slider-crank mechanism with flexibly attached slider

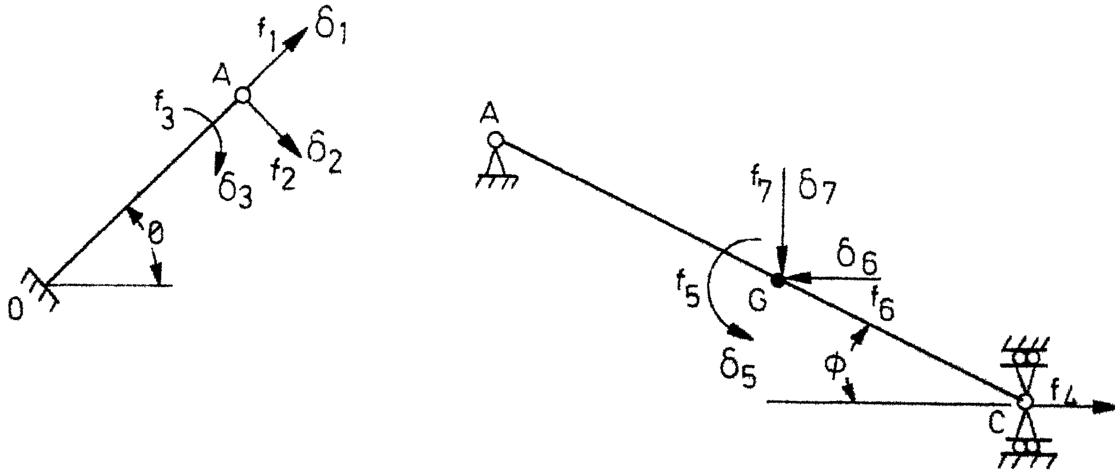

Fig. 2 Figure showing element coordinates (δ) and element forces (f)

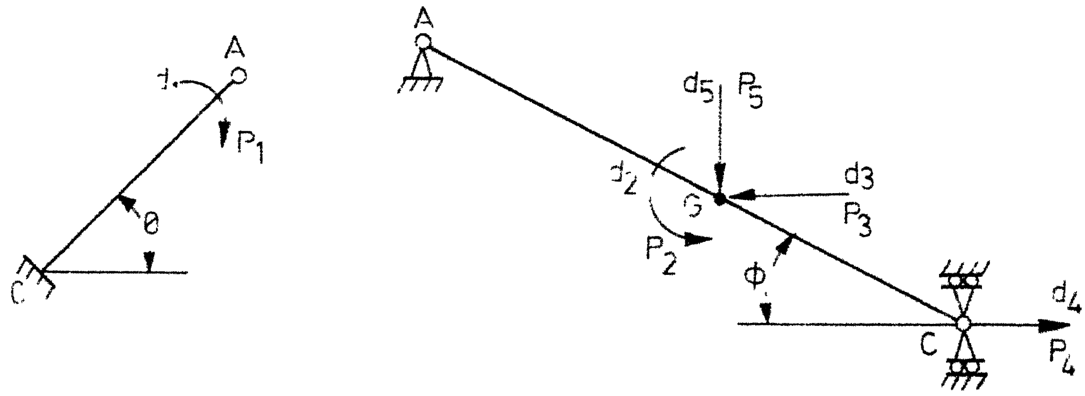

Fig.3 Figure showing system coordinates (d) and system forces (P)

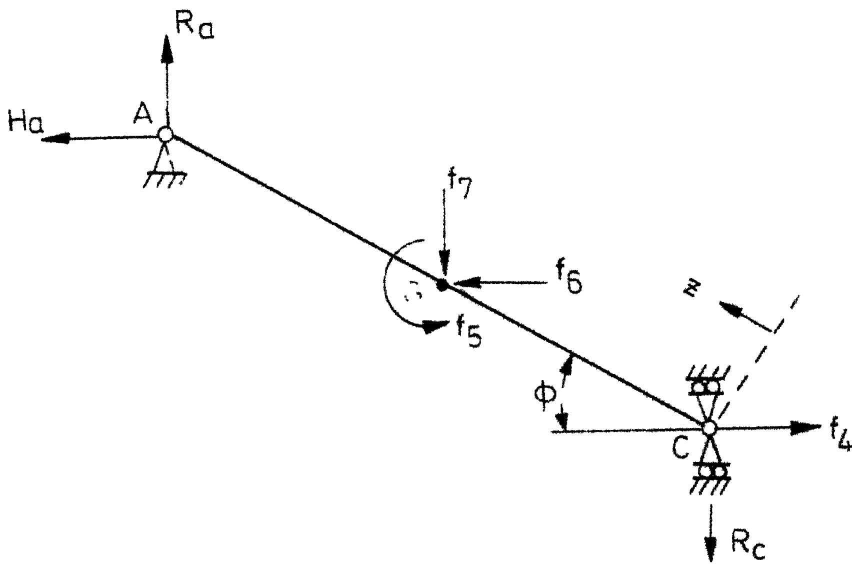

Fig.4 Figure showing free body diagram of the coupler under the action of element forces

phases of the motion. But in the mechanism under consideration the slider is flexibly attached and has mass m_1. Therefore point D is displaced by an amount $(X - X_c)$ with respect to the point C. The displacement X_c would be the algebraic sum of "rigid" displacement and elastic deformation of the coupler end C in x - direction.

2.1.1 The Analysis Procedure :

Since crank and coupler are having mass and inertia and are elastic, it is not possible to obtain displacements, velocities and accelerations of different links at various positions of the crank directly. The displacements would constitute of two components - the dynamic displacements and the elastic deflections. The elastic deflections cannot be obtained directly as they are functions of system forces. For getting 'true' elastic deflections, 'true' system forces should be known. But the 'true' inertia forces depend on 'true' accelerations which in turn depend upon 'true' elastic deflections. Thus, system forces and elastic deflections are inter - dependent and hence a suitable iterative scheme is required to evaluate elastic deflections. The following procedure of analysis is adopted.

The equation of motion of the slider and the energy balance equation (the equation obtained by equating rate of change of total energy to the power input to the system) constitute the equations of analysis. The ordinary differential equations thus

obtained are non - linear and coupled. The values of velocities and accelerations of centre of gravity of the coupler and displacement of the coupler - end C (Fig. 1) are required in order to solve them to yield the velocities and accelerations of the crank and slider, and the slider displacement. Since the required quantities themselves are functions of velocity and acceleration of the crank, these equations cannot be solved directly. An iterative scheme is adopted to solve them.

Firstly, all the velocity and acceleration terms required to solve the above two equations are kinematically expressed in terms of velocity and acceleration of the crank. The displacement of the coupler - end C is also expressed kinematically. The equations are then solved together using fourth order Runge - Kutta method, yielding dynamic velocities and accelerations of the crank and the slider, and the slider displacement. The system forces are then obtained. Elastic analysis via flexibility approach yields element deflections. Then the velocities and accelerations of elastic deformations may be obtained. Knowing accelerations of deformation the dynamic accelerations are modified. The modified system forces, element deflections etc. may now be obtained and the process is iterated. This scheme is iterated to desired accuracy so that system forces, elastic deformations and crank accelerations are best modified.

Now the modified values of velocities and accelerations of centre of gravity of the coupler and coupler - end displacement

may be obtained. With these modified values the differential equations of analysis are again solved to give a better approximation for the velocities and accelerations of the crank and the slider and the slider displacement. The entire procedure is now iterated till the mechanism characteristics obtained from two consecutive iterations match to the desired accuracy.

2.1.2 Derivation of Equations of Analysis

Refering to Fig. 1 the differential equation of motion of the slider may be written as

$$m_1 \ddot{X} + k\,(X - X_c) = 0$$

Or
$$\ddot{X} + w_n^2 X = w_n^2 X_c \qquad (2.1)$$

where $w_n = \sqrt{\dfrac{k}{m_1}}$

From kinematic considerations, one gets

$$\text{Cos } \phi = \left[1 - \left\{ \frac{a}{l} + \frac{r}{l} \text{ Sin } \theta \right\}^2 \right]^{1/2}$$

and
$$\bar{X}_c = r \text{ Cos } \theta + l \left[1 - \left\{ \frac{r}{l} \text{ Sin } \theta + \frac{a}{l} \right\}^2 \right]^{1/2}$$

where

r = radius of crank

l = length of connecting rod

a = amount of offset

θ = angle of the crank with x - axis

ϕ = angle between the coupler and x - axis

$\bar{X}_c$ = "rigid" or kinematic displacement of coupler end C.

It has been shown [26] that if the sum of $\frac{r}{l}$ and $\frac{a}{l}$ is small (e.g. less than 0.3)

$$\left[1 - \left(\frac{r}{l}\ \mathrm{Sin}\ \theta + \frac{a}{l}\right)^2\right]^{1/2} \simeq \left[1 - \frac{1}{2}\left(\frac{r}{l}\ \mathrm{Sin}\ \theta + \frac{a}{l}\right)^2\right]$$

with an error of nearly 0.08%.

Therefore, $\bar{X}_c$ can be rewritten as

$$\bar{X}_c = r\ \mathrm{Cos}\ \theta + l\left[1 - \frac{1}{2}\left\{\frac{r}{l}\ \mathrm{Sin}\ \theta + \frac{a}{l}\right\}^2\right] \qquad (2.2)$$

The working section of the torque - speed characteristics of a three phase a/c motor may be successfully approximated by a parabolic equation when referred to the main shaft [26] .

$$T = a_1 - a_2\ \dot{\theta}^2 \qquad (2.3)$$

where

T = external torque overcoming fluctuations in speed referred to the main shaft.

a_1 & a_2 = Coefficients of applied torque

$\dot{\theta}$ = Crank velocity

Applying the Energy Balance to the mechanism system yields

$$d\ (T.. E.) = T\ d\ \theta \qquad (2.4)$$

Where, T.E. is the total energy of the system at any phase and includes the kinetic energy of crank, coupler and slider mass, the

potential energy stored in the spring and the Strain Energy **stored** in crank and coupler owing to elastic deformations. But due to the presence of a spring, the strain energy stored in the crank and the coupler will be much smaller in comparision with the kinetic and potential energies and hence is not accounted.

Therefore,

$$T.E. = K.E. + P.E.$$

where,

$K.E.$ = kinetic energy of the mechanism

$P.E.$ = Potential energy stored in the spring

Substituting in equation (2.4) results

$$d\,(K.E. + P.E.) = T\,d\,\theta$$

Or

$$\frac{d}{d\theta}(K.E.) + \frac{d}{d\theta}(P.E.) = T = a_1 - a_2\,\dot{\theta}^2 \qquad (2.5)$$

The K.E. of the mechanism is given by

$$K.E. = \frac{1}{2}\left[I_o\,\dot{\theta}^2 + I_G\,\dot{\phi}^2 + m_2\,\dot{X}_G^2 + m_2\,\dot{Y}_G^2 + m_1\,\dot{x}^2 \right] \qquad (2.6)$$

where

I_o = Moment of inertia of crank about the axis of rotation

I_G = Moment of inertia of coupler about its centre of gravity

$\dot{\phi}$ = Angular velocity of rotation of the coupler

$\dot{X}_G$ and $\dot{Y}_G$ = x and y - directional velocities of centre of gravity of the coupler

$\dot{X}$ = Linear velocity of the slider

m_1 = Mass of the slider

m_2 = Mass of the coupler

Differentiating equation (2.6) with respect to θ and setting $Y = \dot{\theta}^2$, one gets

$$\frac{d}{d\theta}(K.E.) = \frac{1}{2} I_o Y' + \frac{1}{\sqrt{Y}} \left[I_G \dot{\phi} \ddot{\phi} + m_2 (\dot{X}_G \ddot{X}_G + \dot{Y}_G \ddot{Y}_G) \right] + \frac{1}{2} m_1 \left[X'^2 Y' + 2Y X' X'' \right] \tag{2.7}$$

Since

$$\frac{d}{d\theta}(\dot{X}^2) = X'^2 \frac{dY}{d\theta} + 2Y X' X''$$

and

$$\frac{d}{d\theta}(\dot{\phi}^2) = \frac{2 \dot{\phi} \ddot{\phi}}{\dot{\theta}} \quad \text{and so on.}$$

The primed quantities in the above equations represent derivatives with respect to θ .

The potential energy stored in the spring is given by

$$P.E. = \frac{1}{2} k (X - X_C)^2 \tag{2.8}$$

Differentiating (2.8) with respect to θ, one gets

$$\frac{d}{d\theta}(P.E.) = k (X - X_C) (X' - X_C') \tag{2.9}$$

Substituting (2.7) and (2.9) in (2.5) and simplifying, one gets

$$Y' \left[\frac{I_o}{2} + \frac{m_1 X'^2}{2} \right] = a_1 - Y (a_2 + m_1 X' X'') - \frac{1}{\sqrt{Y}} \left[I_G \dot{\phi} \ddot{\phi} + m_2 (\dot{X}_G \ddot{X}_G + \dot{Y}_G \ddot{Y}_G) \right] - k (X - X_C) (X' - X_C') \tag{2.10}$$

Since

$$\ddot{X} = \dot{\theta}^2 X'' + \ddot{\theta} X' ,$$

equation (2.1) yields

$$X'' = \frac{1}{Y} \left(w_n^2 (X_C - X) - \frac{X' Y'}{2} \right) \tag{2.11}$$

From (2.11) and (2.10), one can get

$$Y' = \frac{2}{I_o} \Bigg[a_1 - a_2 Y - \frac{1}{\sqrt{Y}} \left\{ I_G \dot{\phi} \ddot{\phi} + m_2 (\dot{X}_G \ddot{X}_G + \dot{Y}_G \ddot{Y}_G) \right\} - k (X - X_C) (X' - X_C') + m_1 X' w_n^2 (X - X_C) \Bigg] \tag{2.12}$$

Equations (2.11) and (2.12) constitute the equations of analysis and are two non - linear coupled ordinary differential equations whose solution would give the values of X, $\dot{X}$, $\dot{\theta}$, $\ddot{X}$ and $\ddot{\theta}$.

2.1.3 Elastic Deformation Of The Mechanism

Any mechanism may be considered a structure if its rigid - body - kinematic degrees of freedom are removed. At every phase the mechanism is "frozen" by removing its rigid - body - kinematic degrees of freedom and then its analysis as a structure yields the elastic deformations.

A mechanism is composed of various combinations of elements each of which can be represented by a known structural model. The deflections of the entire mechanism system may be derived by performing an elastic analysis using flexibility approach. The mechanism will have system, or generalized external forces acting upon it

which will be represented by the column matrix $\left[P_j\right]$ $j = 1, \ldots, m$, where m is the number of system forces. The number of the elastic degrees of freedom n of the system is the sum of elastic degrees of freedom of its elements, each degree of freedom being represented by an element coordinate. For the mechanism under consideration the number of elastic degrees of freedom n is taken to be 7 while 5 generalized forces act on the mechanism. (Fig. 2 and Fig. 3)

2.1.3.1 Matrix Of Element Flexibilities

The mechanism constituting of crank and coupler only is to be analyzed for elastic deformation at each phase. It has got only one rigid - body - kinematic degree of freedom. Therefore, it may be converted into a structure by modeling the input link as a cantilever or fixed - free beam. As the end C of the coupler is assumed to move along x - axis only, the coupler is modeled as a beam hinged at the end A and having double roller support at the end C as shown in Figure 2.

The element flexibility matrix $\left[F_{cr}\right]$ of the crank is given by [8]

$$\left[F_{cr}\right] = \begin{bmatrix} r/A_o E_1 & 0 & 0 \\ 0 & r^3/3E_1 I_1 & r^2/2E_1 I_1 \\ 0 & r^2/2E_1 I_1 & r/E_1 I_1 \end{bmatrix} \qquad (2.13)$$

where

A_o = Area of cross - section of the crank

E_1 = Modulus of elasticity of the material of crank

I_1 = Section - modulus of the crank.

The element flexibility matrix of the coupler can be constructed by using Castigliano's Theorem.

Refering to FIG. 4 the strain energy per unit volume stored in the material of coupler is given by

$$U = \frac{1}{2}\left[\int_{0}^{1/2} \frac{(f_4 \cos\phi + R_C \sin\phi)^2}{E_2 A_G}\, dz + \int_{1/2}^{1} \frac{(f_4 \cos\phi + R_C \sin\phi - f_6 \cos\phi + f_7 \sin\phi)^2}{E_2 A_G}\, dz + \int_{0}^{1/2} \frac{(R_C \cos\phi - f_4 \sin\phi)^2}{E_2 I_2} z^2\, dz + \int_{1/2}^{1} \frac{(-f_4 z \sin\phi + R_C z \cos\phi + f_6 (z - \frac{1}{2}) \sin\phi + f_7 (z - \frac{1}{2}) \cos\phi - f_5)^2}{E_2 I_2}\, dz\right] \quad (2.14)$$

Where

A_G = Area of cross - section of the coupler

E_2 = Modulus of elasticity of the coupler

I_2 = Section - modulus of the coupler

R_C = Reaction at C

R_C, the reaction at the point C, can be evaluated by taking moment of all the element forces about A. Then one gets

$$R_c = -\frac{f_6}{2}\tan\phi + \frac{f_5}{1 \cos\phi} + f_4 \tan\phi - \frac{f_7}{2} \quad (2.15)$$

Now applying Castigliano's Theorem, the differentiation of equation (2.14) with respect to f_4 results

$$\delta_4 = \frac{\partial U}{\partial f_4} = \left[\int_0^{1/2} \frac{(f_4 \, \mathrm{Cos}\, \phi + R_C \, \mathrm{Sin}\, \phi)}{E_2 \, A_G} \, \frac{dz}{\mathrm{Cos}\, \phi} \right.$$

$$\left. + \int_{1/2}^{1} \frac{(f_4 \, \mathrm{Cos}\, \phi + R_C \, \mathrm{Sin}\, \phi - f_6 \, \mathrm{Cos}\, \phi + f_7 \, \mathrm{Sin}\, \phi)}{E_2 \, A_G} \, \frac{dz}{\mathrm{Cos}\, \phi} \right]$$

which results after simplification as

$$\delta_4 = f_4 \, \frac{1 \, \mathrm{Sec}^2 \phi}{E_2 \, A_G} + f_5 \, \frac{\tan \phi \, \mathrm{Sec}\, \phi}{E_2 \, A_G} - f_6 \, \frac{1 \, \mathrm{Sec}^2 \phi}{2 \, E_2 \, A_G} \tag{2.16}$$

Differentiation of equation (2.14) with respect to f_5 results

$$\delta_5 = \frac{\partial U}{\partial f_5} = \left[\int_0^{1/2} \frac{(f_4 \, \mathrm{Cos}\, \phi + R_C \, \mathrm{Sin}\, \phi)}{E_2 \, A_G} \, \frac{\tan \phi}{1} \, dz \right.$$

$$+ \int_{1/2}^{1} \frac{(f_4 \, \mathrm{Cos}\, \phi + R_C \, \mathrm{Sin}\, \phi - f_6 \, \mathrm{Cos}\, \phi + f_7 \, \mathrm{Sin}\, \phi)}{E_2 \, A_G} \, \frac{\tan \phi}{1} \, dz$$

$$+ \int_0^{1/2} \frac{(R_C \, \mathrm{Cos}\, \phi - f_4 \, \mathrm{Sin}\, \phi)}{E_2 \, I_2} \, \frac{z^2}{1} \, dz$$

$$+ \int_{1/2}^{1} \frac{(-f_4 \, z \, \mathrm{Sin}\, \phi + R_C \, z \, \mathrm{Cos}\, \phi + f_6 \, (z - \frac{1}{2}) \, \mathrm{Sin}\, \phi + f_7 \, (z - \frac{1}{2}) \mathrm{Cos}\, \phi - f_5)}{E_2 \, I_2}$$

$$\left. \left(\frac{z}{1} - 1 \right) dz \right]$$

which, on simplification, can be written as

$$\delta_5 = f_4 \frac{\text{Sec}\,\phi\ \tan\phi}{E_2\ A_G} + f_5 \left[\frac{1}{12\ E_2\ I_2} + \frac{\tan^2\phi}{1\ E_2\ A_G}\right] - f_6 \frac{\text{Sec}\,\phi\ \tan\phi}{2\ E_2\ A_G} \tag{2.17}$$

Differentiating equation (2.14) with respect to f_6, one gets

$$\delta_6 = \frac{\partial U}{\partial f_6} = -\int_0^{1/2} \frac{(f_4\ \text{Cos}\,\phi + R_C\ \text{Sin}\,\phi)}{E_2\ A_G} \frac{\text{Sin}\,\phi\ \tan\phi}{2}\,dz$$

$$-\int_{1/2}^{1} \frac{(f_4\ \text{Cos}\,\phi + R_C\ \text{Sin}\,\phi - f_6\ \text{Cos}\,\phi + f_7\ \text{Sin}\,\phi)}{E_2\ A_G}$$

$$(\text{Cos}\,\phi + \frac{\text{Sin}\,\phi\ \tan\phi}{2})\,dz$$

$$+\int_{1/2}^{1} \frac{(-f_4\ z\ \text{Sin}\,\phi + R_C\ z\ \text{Cos}\,\phi + f_6\ (z - \frac{1}{2})\ \text{Sin}\,\phi + f_7\ (z - \frac{1}{2})\text{Cos}\,\phi - f_5)}{E_2\ I_2}$$

$$\frac{(z-1)}{2}\ \text{Sin}\,\phi\ dz - \int_0^{1/2} \frac{(R_C\ \text{Cos}\,\phi - f_4\ \text{Sin}\,\phi)}{E_2\ I_2} \frac{\text{Sin}\,\phi}{2}\ z^2\,dz\ ,$$

Or

$$\delta_6 = -f_4 \frac{1\ \text{Sec}^2\phi}{2\ E_2\ A_G} - f_5 \frac{\text{Sec}\,\phi\ \tan\phi}{2\ E_2\ A_G} + f_6 \left[\frac{1\ (\text{Sec}^2\phi + \text{Cos}^2\phi)}{4\ E_2\ A_G} + \frac{1^3\ \text{Sin}^2\phi}{48\ E_2\ I_2}\right] + f_7 \left[\frac{1^3\ \text{Sin}\ 2\phi}{96\ E_2\ I_2} - \frac{1\ \tan\phi\ (1 + \text{Cos}\ 2\phi)}{8\ E_2\ A_G}\right] \tag{2.18}$$

Differentiating equation (2.14) with respect to f_7, one gets

$$\delta_7 = \frac{\partial U}{\partial f_7} = - \int_0^{l/2} \frac{(f_4 \text{ Cos } \phi + R_C \text{ Sin } \phi)}{E_2 \, A_G} \; \frac{\text{Sin } \phi}{2} \, dz$$

$$+ \int_{l/2}^{l} \frac{(f_4 \text{ Cos } \phi + R_C \text{ Sin } \phi - f_6 \text{ Cos } \phi + f_7 \text{ Sin } \phi)}{E_2 \, A_G} \; \frac{\text{Sin } \phi}{2} \, dz$$

$$- \int_0^{l/2} \frac{(R_C \text{ Cos } \phi - f_4 \text{ Sin } \phi)}{2 \, E_2 \, I_2} \, z^2 \text{ Cos } \phi \, dz$$

$$+ \int_{l/2}^{l} \frac{(- f_4 \, z \text{ sin } \phi + R_C \, z \text{ Cos } \phi + f_6 \, (z - \frac{l}{2}) \text{ Sin } \phi + f_7 \, (z - \frac{l}{2}) \text{Cos } \phi - f_5)}{E_2 \, I_2}$$

$$\frac{(z - l)}{2} \text{ Cos } \phi \, dz$$

Simplification of the above equation results in

$$\delta_7 = f_6 \left[\frac{l^3 \text{ Sin } 2\phi}{96 \, E_2 \, I_2} - \frac{l \text{ Sin } 2\phi}{8 \, E_2 \, A_G} \right] + f_7 \left[\frac{l \text{ Sin}^2 \phi}{4 \, E_2 \, A_G} + \frac{l^3 \text{ Cos}^2 \phi}{48 \, E_2 \, I_2} \right] \tag{2.19}$$

The element deformations are represented by a n - dimensional column matrix $\left[\delta_i\right]$ and are related to the element forces $\left[f_i\right]$ by the relation

$$\vec{\delta} = \left[F_e\right] \vec{f} \tag{2.20}$$

where $\left[f_i\right]$ is n - dimensional column matrix representing the element forces and $\left[F_e\right]$ is the element flexibility matrix (n x n) of the mechanism and can be constructed by using equations (2.13), (2.16), (2.17), (2.18) and (2.19) as

$$\left[F_e \right] = \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_2 & C_3 & 0 & 0 & 0 & 0 \\ 0 & C_3 & C_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_5 & C_6 & C_7 & 0 \\ 0 & 0 & 0 & C_6 & C_8 & C_9 & 0 \\ 0 & 0 & 0 & C_7 & C_9 & C_{10} & C_{11} \\ 0 & 0 & 0 & 0 & 0 & C_{11} & C_{12} \end{bmatrix} \tag{2.21}$$

where

$$C_1 = \frac{r}{A_o E_1}$$

$$C_2 = \frac{r^3}{3 E_1 I_1}$$

$$C_3 = \frac{r^2}{2 E_1 I_1}$$

$$C_4 = \frac{r}{E_1 I_1}$$

$$C_5 = \frac{l \operatorname{Sec}^2 \phi}{E_2 A_G}$$

$$C_6 = \frac{\tan \phi \operatorname{Sec} \phi}{E_2 A_G}$$

$$C_7 = -\frac{l \operatorname{Sec}^2 \phi}{2 E_2 A_G}$$

$$C_8 = \frac{l}{12 E_2 I_2} + \frac{\tan^2 \phi}{l E_2 A_G}$$

$$C_9 = -\frac{\operatorname{Sec} \phi \tan \phi}{2 E_2 A_G}$$

$$C_{10} = \frac{l\ (\mathrm{Sec}^2\ \phi + \mathrm{Cos}^2\ \phi)}{4\ E_2\ A_G} + \frac{l^3\ \mathrm{Sin}^2\ \phi}{48\ E_2\ I_2}$$

$$C_{11} = \frac{l^3\ \mathrm{Sin}\ 2\ \phi}{96\ E_2\ I_2} - \frac{l\ \mathrm{Sin}\ 2\ \phi}{8\ E_2\ A_G}$$

$$C_{12} = \frac{l\ \mathrm{Sin}^2\ \phi}{4\ E_2\ A_G} + \frac{l^3\ \mathrm{Cos}^2\ \phi}{48\ E_2\ I_2} \tag{2.22}$$

2.1.3.2 Force Transfer Matrix

In order to transfer the system forces $\left[P_j\right]$ $j = 1, \ldots, m$ into element or internal forces $\left[f_i\right]$ $i = 1, \ldots, n$ an $n \times m$ force transfer matrix $\left[D\right]$ is derived by the methods of static analysis.

The transformation relation is given by

$$\vec{f} = \left[D\right] \vec{P} \tag{2.23}$$

The force transfer matrix $\left[D\right]$ also relates the system deformations represented by m - dimensional column matrix $\left[d_j\right]$ and element deformation $\left[\delta_i\right]$ and the relation is given by

$$\vec{d} = \left[D\right]^T \vec{\delta} \tag{2.24}$$

Equations (2.20), (2.23) and (2.24) can be used to get the relationship between system deformations $\left[d_j\right]$ and system forces $\left[P_j\right]$. Thus substitution of (2.20) and (2.23) in (2.24) results in

$$\vec{d} = \left[D\right]^T \left[F_e\right] \left[D\right] \vec{P} \tag{2.25}$$

or

$$\vec{d} = \left[F_s\right] \vec{P} \tag{2.26}$$

where

$$\left[F_s\right] = \left[D\right]^T \left[F_e\right] \left[D\right] \tag{2.27}$$

is a m x m matrix and is known as system flexibility matrix.

The force transfer matrix $[D]$ can be constructed by using the methods of static analysis.

Refering to FIGURES 2, 3 and 4 representing force diagrams of the elements, one gets

$$
\begin{aligned}
f_3 &= P_1 \\
f_5 &= P_2 \\
f_6 &= P_3 \\
f_4 &= P_4 \\
f_7 &= P_5
\end{aligned}
\qquad (2.28)
$$

$$
\begin{aligned}
f_1 &= H_a \text{ Cos } \theta - R_a \text{ Sin } \theta \\
f_2 &= H_a \text{ Sin } \theta + R_a \text{ Cos } \theta
\end{aligned}
\qquad (2.29)
$$

The system forces can be written in terms of accelerations as

$$
\begin{aligned}
P_1 &= I_o \ddot{\theta} = \frac{1}{2} I_o Y' \\
P_2 &= I_G \ddot{\phi} \\
P_3 &= m_2 \ddot{X}_G \\
P_4 &= k (X - X_C) \\
P_5 &= m_2 \ddot{Y}_G
\end{aligned}
\qquad (2.30)
$$

The reactions H_a and R_a can be obtained using free- body - diagram of the coupler FIG. 4. Thus,

$$H_a = f_4 - f_6$$

$$\text{and } R_a = -\frac{f_6}{2} \tan \phi + \frac{f_5}{1} \cos \phi + f_4 \tan \phi + \frac{f_7}{2} \tag{2.31}$$

Substituting (2.31) in (2.29) and simplifying, one gets

$$f_1 = -P_2 \frac{\sin \theta}{1 \cos \phi} + P_3 \left(\frac{\sin \theta \tan \phi}{2} - \cos \theta \right)$$

$$+ P_4 \ (\cos \theta - \tan \phi \sin \theta) - \frac{P_5}{2} \sin \theta$$

and

$$f_2 = P_2 \frac{\cos \theta}{1 \cos \phi} - P_3 \left(\sin \theta + \frac{\cos \theta \tan \phi}{2} \right)$$

$$+ P_4 \ (\sin \theta + \tan \phi \cos \theta) + \frac{P_5}{2} \cos \theta \tag{2.32}$$

from (2.28) and (2.32), one can obtain that

$$\left[D \right] = \begin{bmatrix} 0 & D_1 & D_2 & D_3 & D_4 \\ 0 & D_5 & D_6 & D_7 & D_8 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.33}$$

where,

$$D_1 = -\frac{\sin \theta}{1 \cos \phi}$$

$$D_2 = \frac{\sin \theta \tan \phi}{2} - \cos \theta$$

$$D_3 = \cos \theta - \tan \phi \sin \theta$$

$$D_4 = -\frac{\sin \theta}{2}$$

$$D_5 = \frac{\text{Cos } \theta}{1 \text{ Cos } \phi}$$

$$D_6 = -\left(\text{Sin } \theta + \frac{\text{Cos } \theta \tan \phi}{2}\right)$$

$$D_7 = \text{Sin } \theta + \tan \phi \text{ Cos } \theta$$

$$D_8 = \frac{\text{Cos } \theta}{2} \qquad (2.34)$$

2.1.3.3 System Flexibility Matrix

Finally the system flexibility matrix, an (m x m) matrix, may be derived by substituting equations (2.21) and (2.33) in (2.27).

$$\left[F_s\right] = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 \\ F_2 & F_6 & F_7 & F_8 & F_9 \\ F_3 & F_7 & F_{10} & F_{11} & F_{12} \\ F_4 & F_8 & F_{11} & F_{13} & F_{14} \\ F_5 & F_9 & F_{12} & F_{14} & F_{15} \end{bmatrix} \qquad (2.35)$$

where

$$F_1 = C_4$$

$$F_2 = C_3 D_5$$

$$F_3 = C_3 D_6$$

$$F_4 = C_3 D_7$$

$$F_5 = C_3 D_8$$

$$F_6 = C_1 D_1^2 + C_2 D_5^2 + C_8$$

$$F_7 = C_1 D_1 D_2 + C_2 D_5 D_6 + C_9$$

$$F_8 = C_1 D_1 D_3 + C_2 D_5 D_7 + C_6$$

$$F_9 = C_1 D_1 D_4 + C_2 D_5 D_8$$

$$F_{10} = C_1 D_2^2 + C_2 D_6^2 + C_{10}$$

$$F_{11} = C_1 D_2 D_3 + C_2 D_6 D_7 + C_7$$

$$F_{12} = C_1 D_2 D_4 + C_2 D_6 D_8 + C_{11}$$

$$F_{13} = C_1 D_3^2 + C_2 D_7^2 + C_5$$

$$F_{14} = C_1 D_3 D_4 + C_2 D_7 D_8$$

$$F_{15} = C_1 D_4^2 + C_2 D_8^2 + C_{12} \qquad (2.36)$$

2.1.3.4 System deformations and their velocities and accelerations

Having derived the system flexibility matrix, the system or generalized deformations $\left[d_j\right]$ $j = 1, \ldots\ldots, m$ may be obtained directly from equation (2.26)

Refering to FIG. 3 one can conclude that

$$\theta_e = d_1$$

$$\phi_e = d_2$$

$$X_{G_e} = d_3$$

$$X_{C_e} = d_4$$

$$Y_{G_e} = d_5 \qquad (2.37)$$

where suffix e denotes the elastic deformations (e.g. X_{C_e} is the elastic deformation at point C of the coupler in x - direction).

The velocities and accelerations of deformation may then be obtained by using finite difference technique. Thus, one may derive

$$
\begin{aligned}
\dot{\theta}_e &= \theta_e' \, \dot{\theta} \\
\dot{\phi}_e &= \phi_e' \, \dot{\theta} \\
\dot{X}_{G_e} &= X_{G_e}' \, \dot{\theta} \\
\dot{Y}_{G_e} &= Y_{G_e}' \, \dot{\theta} \\
\dot{X}_{C_e} &= X_{C_e}' \, \dot{\theta}
\end{aligned}
\tag{2.38}
$$

and

$$
\begin{aligned}
\ddot{\theta}_e &= \theta_e' \, \ddot{\theta} + \theta_e'' \, \dot{\theta}^2 = \theta_e' \, \ddot{\theta} + \theta_e'' \, Y \\
\ddot{\phi}_e &= \phi_e' \, \ddot{\theta} + \phi_e'' \, Y \\
\ddot{X}_{G_e} &= X_{G_e}' \, \ddot{\theta} + X_{G_e}'' \, Y \\
\ddot{Y}_{G_e} &= Y_{G_e}' \, \ddot{\theta} + Y_{G_e}'' \, Y \\
\ddot{X}_{G_c} &= X_{C_e}' \, \ddot{\theta} + X_{C_e}'' \, Y
\end{aligned}
\tag{2.39}
$$

2.2 Solution Technique

After having mathematically formulated the problem, the next task is to devise some iterative technique to solve the equations. As discussed earlier the equations of analysis cannot be solved directly.

Refering to equations (2.11) and (2.12), one may conclude that this set of two non - linear coupled ordinary differential

equations may be solved directly to yield the values of X, $\dot{X}$, $\dot{\theta}$, $\ddot{X}$, $\ddot{\theta}$ etc. provided the displacement, velocity and acceleration terms (e.g. X_G, $\dot{X}_G$, $\ddot{X}_G$ etc.) appearing on the right hand side of the equations are known. But these displacement, velocity and acceleration terms are not only kinematically related to the unknown input crank velocity $\dot{\theta}$ and acceleration $\ddot{\theta}$, but also depend upon the elastic deformations, velocities and accelerations of deformation. Elastic deformations, velocities and the accelerations of deformation depend upon the inertia forces which in turn depend upon the acceleration terms. Since the acceleration terms and the inertia forces are inter-dependent, this set of equations can be best solved iteratively with the set of equations (2.26), (2.35), (2.37), (2.38) and (2.39).

2.2.1 Iterative Schemes

As stated earlier the equations of analysis alongwith related equations cannot be solved directly and some iterative scheme is to be adopted for their solution. The procedure adopted for their solution is shown on the flow chart (Appendix A). The steps are as follows :

STEP 1 : The "rigid" displacement, velocity and acceleration terms appearing on the right hand side of the equations (2.11) and (2.12) are expressed in terms of velocity and acceleration of the crank. These quantities are then substituted in equations of analysis. To start with, the links are considered to be rigid.

STEP 2 : Equations (2.11) and (2.12) can now be solved using some numerical technique (e.g. fourth order Runge - Kutta Method). Since the above equations constitute a boundary - value problem, following procedure is adopted for their solution.

(i) Following values are assumed in the first step.

(a) Initial guess values for slider displacement X, its derivative X' with respect to independent parameter θ and initial velocity of the main drive $\dot{\theta}$ (or for $Y = \dot{\theta}^2$).

(b) Applied torque constants a_1 and a_2.

(ii) Using Runge - Kutta method values of X, X' and Y are computed for each 2° of input - crank rotation.

(iii) The initial values of X, X' and Y at $\theta = 0$ and their values at $\theta = 2\pi$ are compared. In the steady state they should be equal, for this denotes that the mechanism maintains an average speed in operation. If $X (\theta = 0) \neq X (\theta = 2\pi)$ the initial guess for X at $\theta = 0$ is corrected. Similarly if $X' (\theta = 0) \neq X' (\theta = 2\pi)$ the initial guess value of X' is corrected. If $Y (\theta = 0) \neq Y (\theta = 2\pi)$ then the a_1 term of the torque expression ($T = a_1 - a_2 \dot{\theta}^2$) is suitably changed.

(iv) The average computed value of Y is compared with Ω^2 where Ω is the desired average angular velocity of crank. The average value of Y is obtained as an arithmetic mean of the 180 computed values of Y based on equal step in θ. If average value of Y is not close to Ω^2 the initial guess value of Y is modified.

(v) If the requirements of steps (iii) and (iv) are not satisfied, with the modified values of X, X', Y and a_1 at $\theta = 0$ the equations (2.11) and (2.12) are again solved. The iterative scheme is continued till requirements of steps (iii) and (iv) are met.

STEP 3 : Having solved equations (2.11) and (2.12) the initial position of the mechanism ($\theta = 0$) is set.

STEP 4 : Solution of step (2) yields slider displacement, velocities and accelerations of the slider and the crank. Knowing accelerations the system forces are evaluated using expressions (2.30).

STEP 5 : Elastic deformations may, then, be computed using expressions (2.26).

STEP 6 : Using expressions (2.38 and 2.39) velocities and accelerations of deformation are computed.

STEP 7 : The values of acceleration terms are modified using results of step (6).

STEP 8 : Values of new and old accelerations are compared. If the values do not match to the desired accuracy the entire procedure from step (4) and onwards is repeated. Finally the modified values of inertia forces and accelerations are obtained.

STEP 9 : All the velocity terms are modified using results of step (6). Displacement of coupler end C is also modified using results of step (5).

STEP 10 : The process from step (4) to step (9) is repeated for all mechanism positions between 0 and 2π with a step increment of 2°.

STEP 11 : If the procedure from step (2) to step (10) is iterated once only, step (2) is again undertaken with modified displacements, velocities and accelerations and the procedure is repeated.

STEP 12 : New and old values of slider displacement X are compared. If the values match to the desired accuracy the process is terminated. Otherwise the entire procedure from step (2) to step (11) is iterated.

CHAPTER III

RESULTS AND DISCUSSIONS

To illustrate the application of the mathematical model and solution technique developed in Chapter II three numerical examples have been worked out. All the three examples were taken from the published literature so that the results obtained may be compared with the existing ones. In the first example problem the mechanism considered by Davidson [5] was studied. In other two example problems the mechanisms synthesized by Liniecki [26] were worked out.

3.1 Example 1 :

The mechanism studied in this example problem is schematically identical with that examined by Davidson [5]. In Davidson's model the radius of crank, length of connecting rod and the amount of offset are taken to be 2.68 in., 9.38 in. and 0.0 in. respectively. The driving crank rotates at an average speed of 200 rev/min and the slider weight is taken as 4 lb. wt.. The stiffness of the spring is taken to be 42.5 lb/in.. The masses and elasticity of the crank and the connecting rod are neglected in his model. Whereas in the present dissertation they are taken into account. The crank is assumed to have an area of cross - section of 1.0 sq. in. and moment of inertia about its axis of rotation as 1.0 lb-in-sec^2. The coupler weighs 2.0 lb. and has an area of cross - section as 0.75 in.^{2}. Its moment

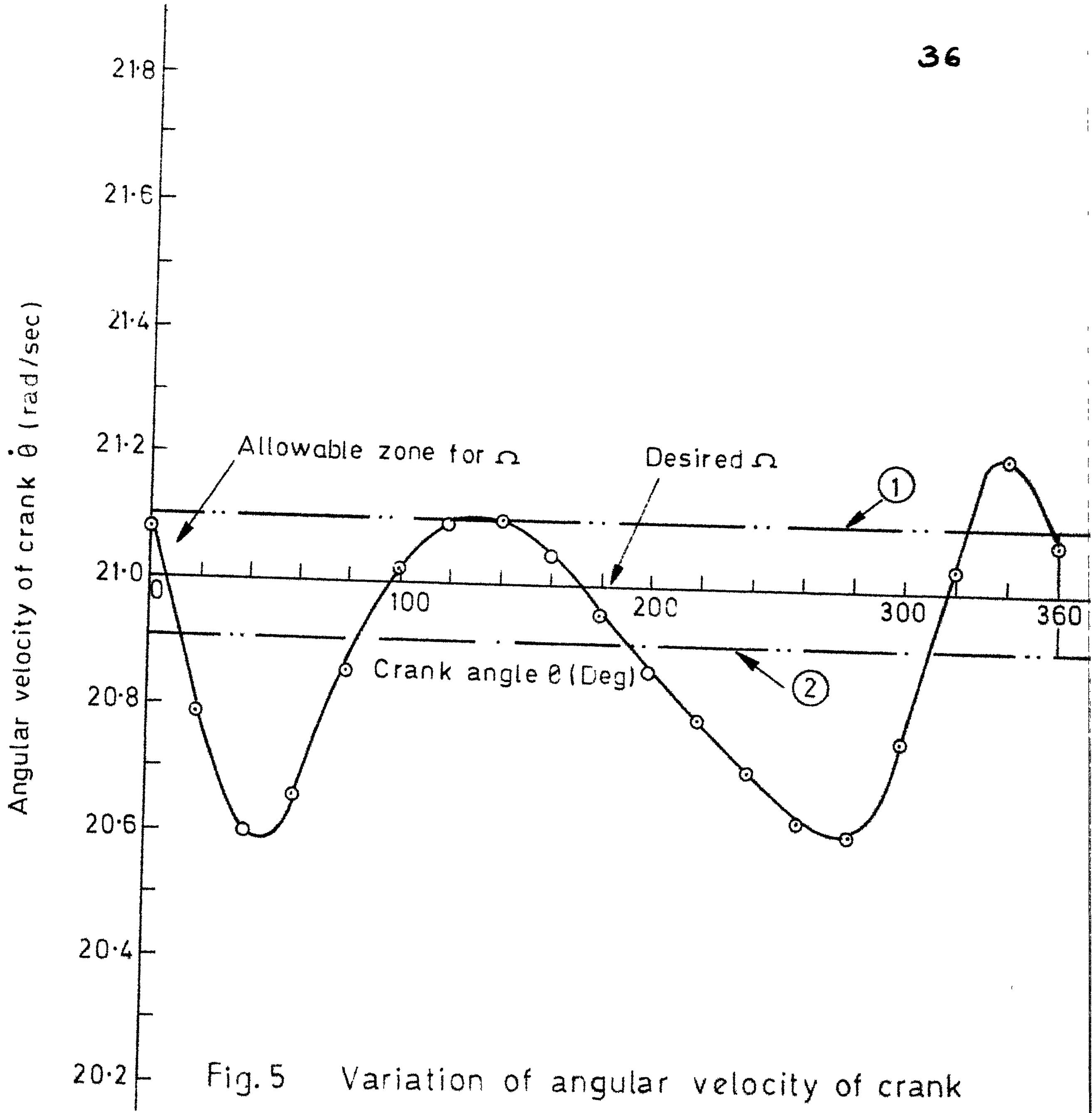

Fig. 5 Variation of angular velocity of crank with respect to crank angle

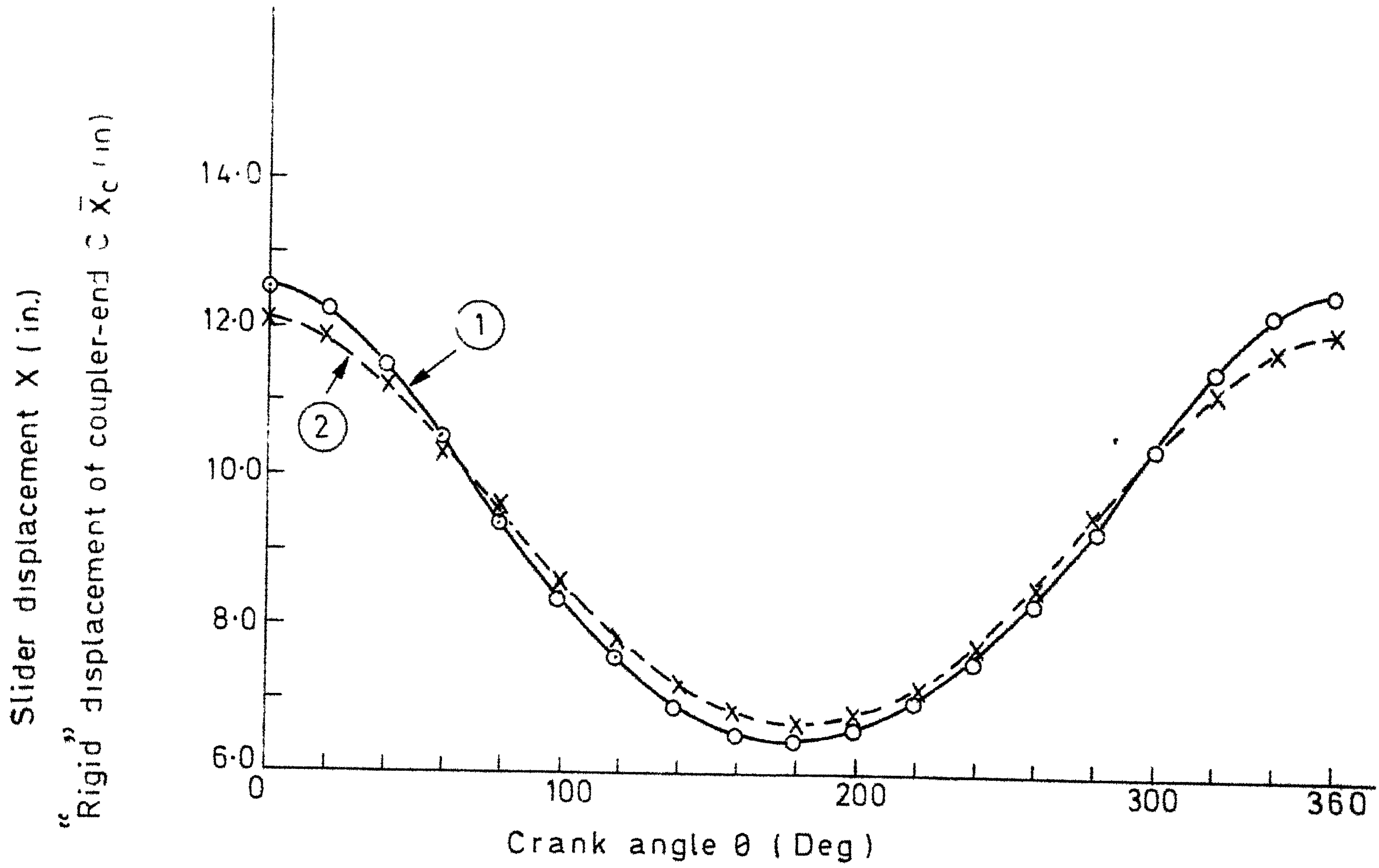

Fig. 6 Variation of slider displacement and "Rigid" displacement of coupler-end C with respect to crank angle

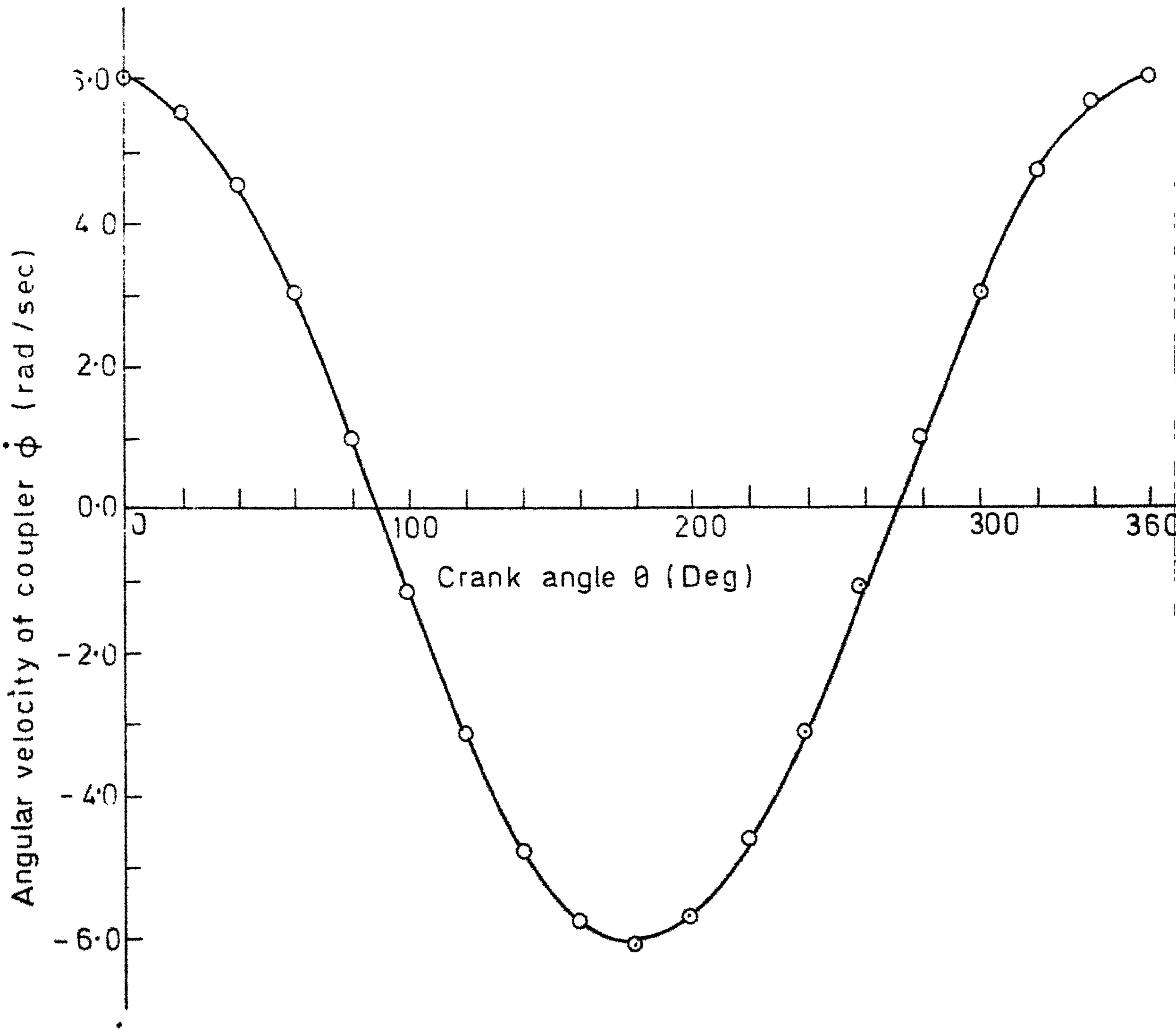

Fig.7 Variation of angular velocity of coupler with respect to crank angle

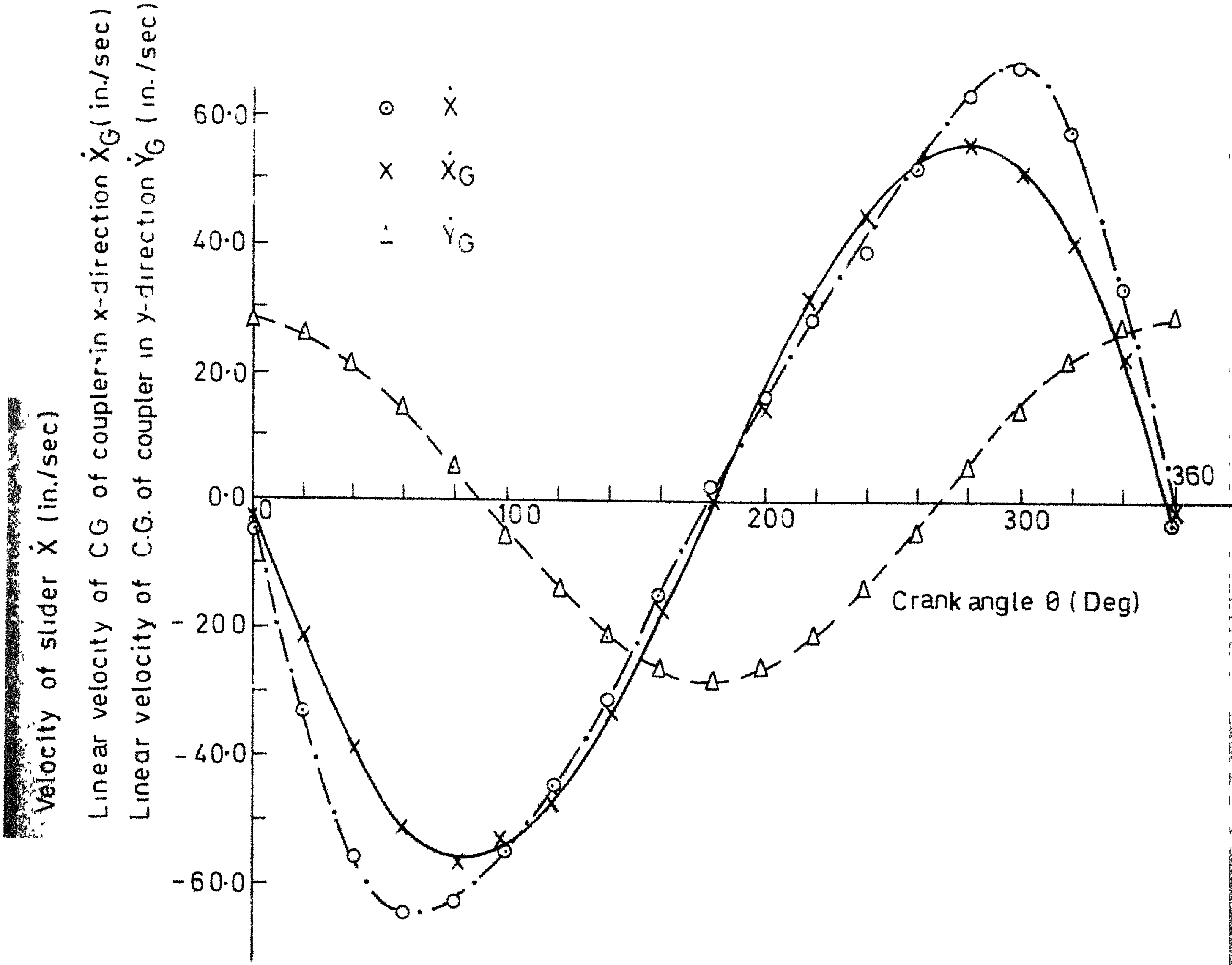

Fig. 8 Variation of velocities of slider & centre of gravity of coupler in x and y-directions with respect to crank angle

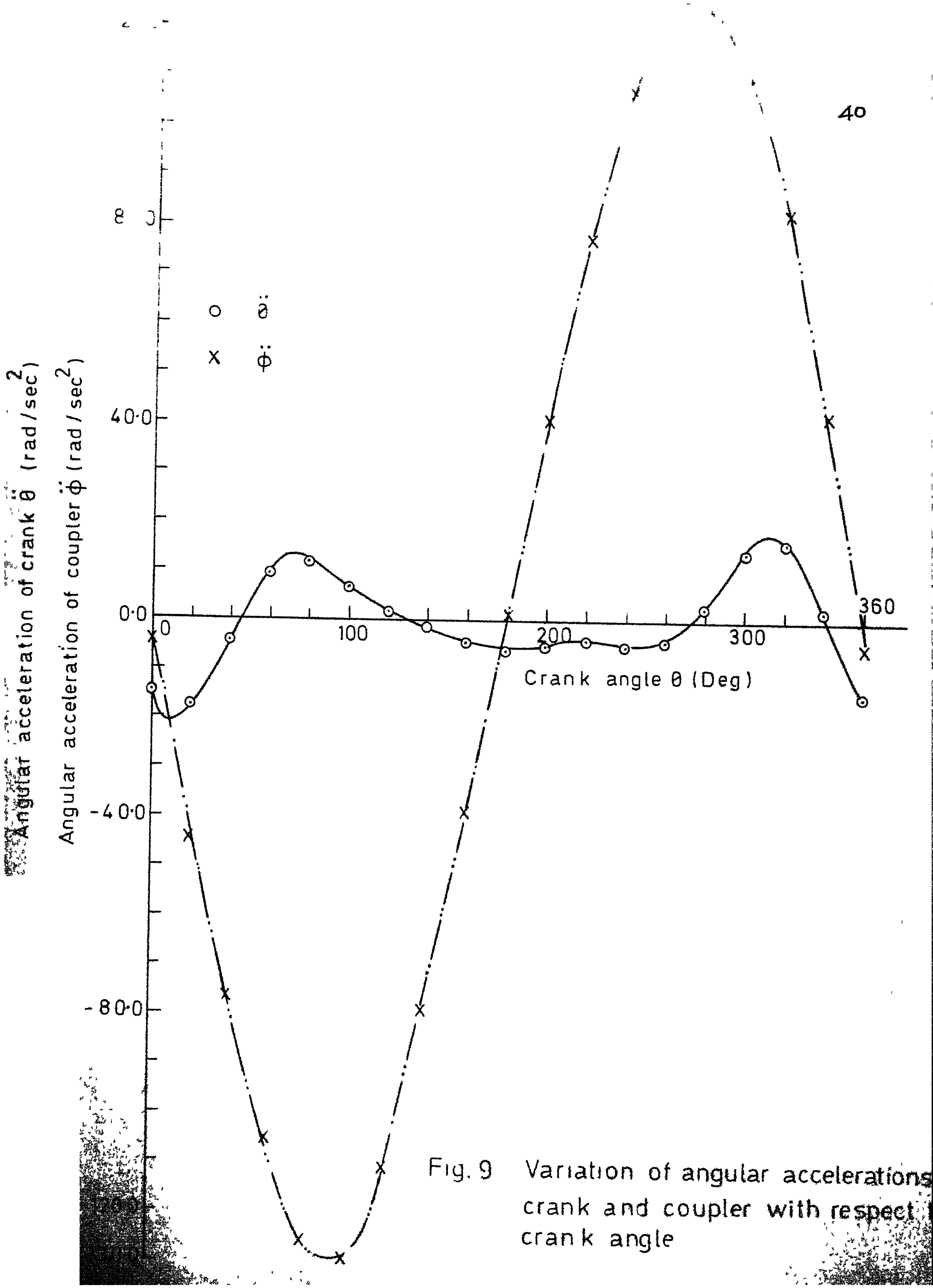

Fig. 9 Variation of angular accelerations of crank and coupler with respect to crank angle

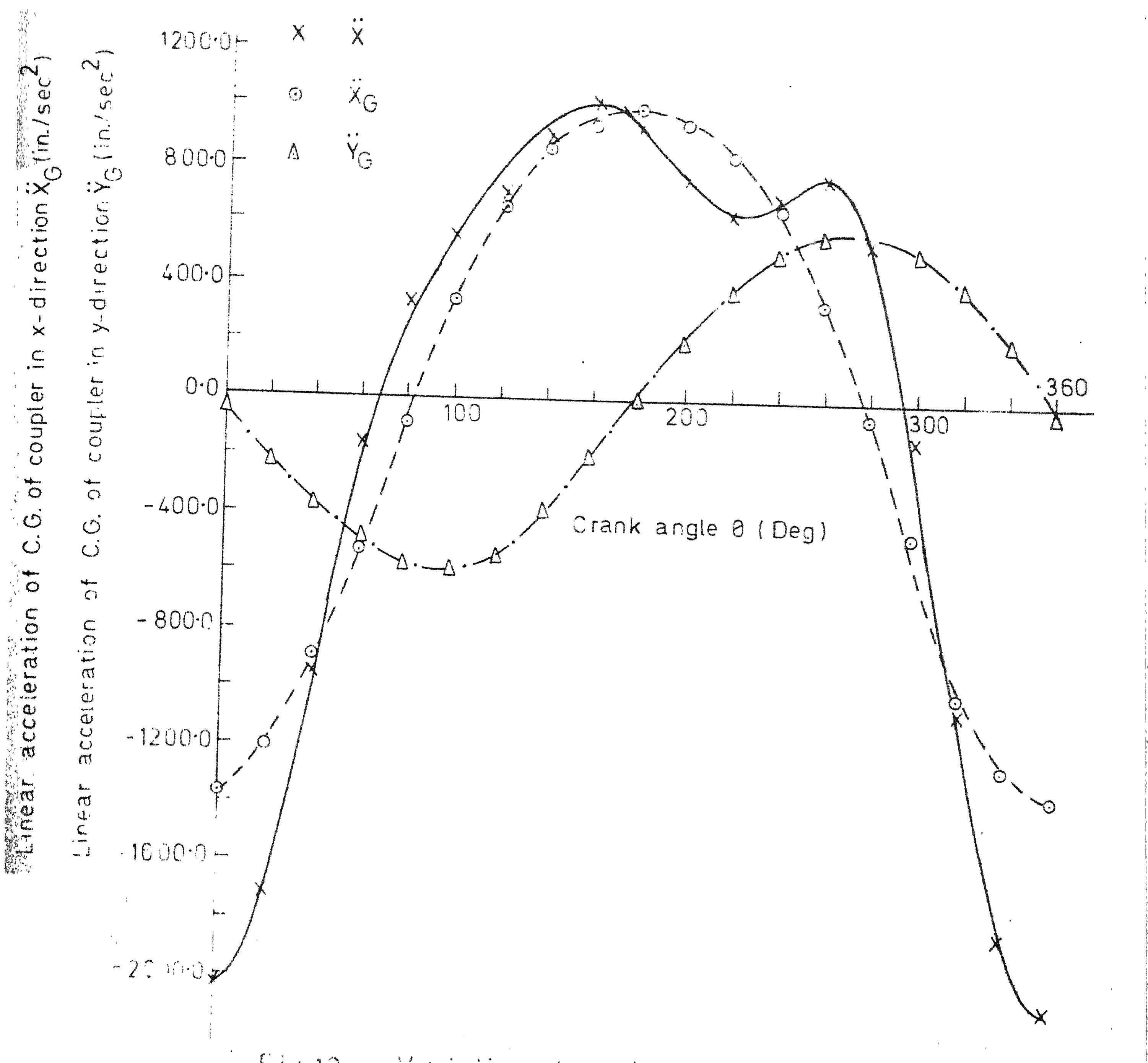

Fig.10 Variation of accelerations of slider and centre of gravity of coupler in x and y-directions with respect to crank angle

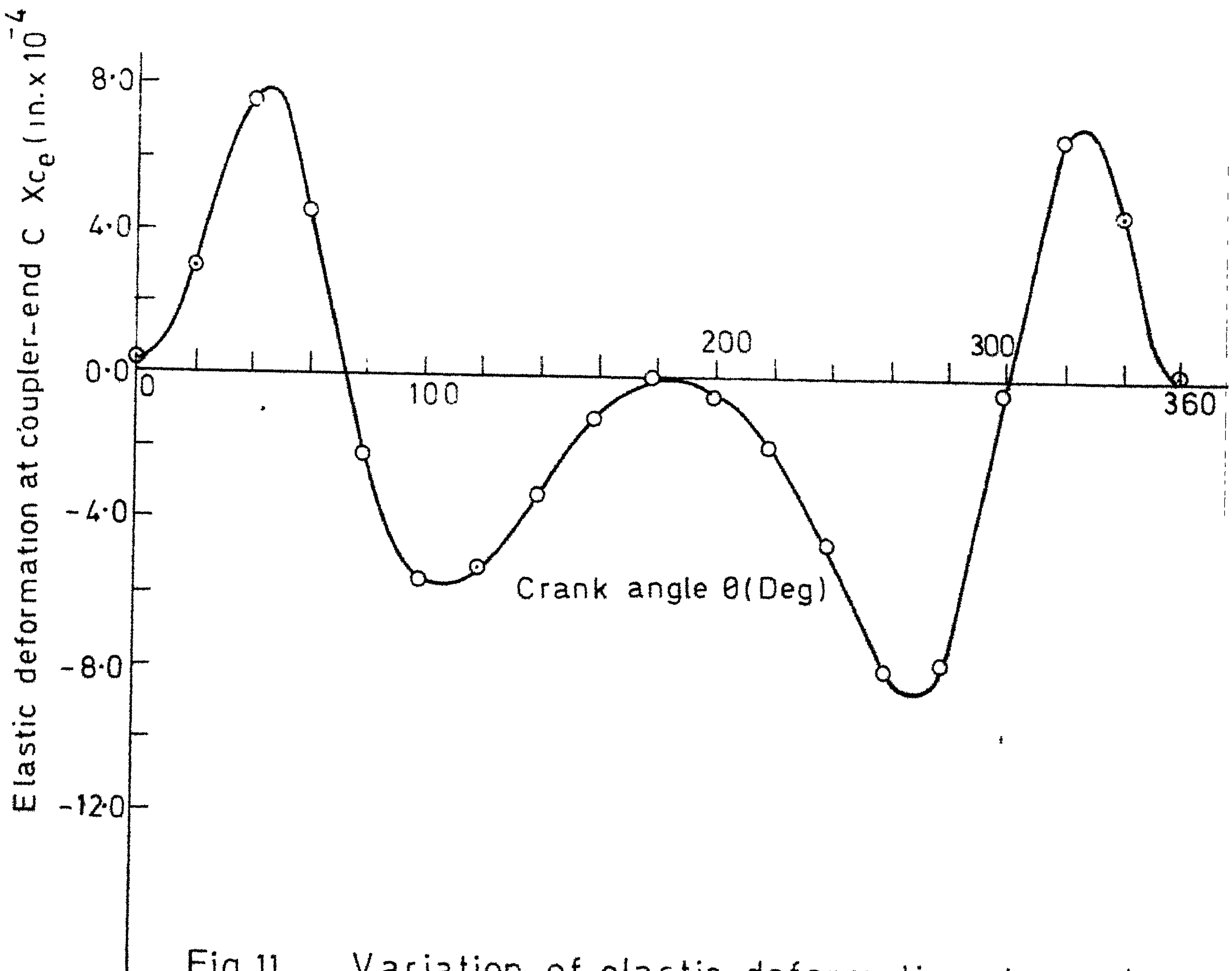

Fig. 11 Variation of elastic deformation at coupler end C with respect to crank angle

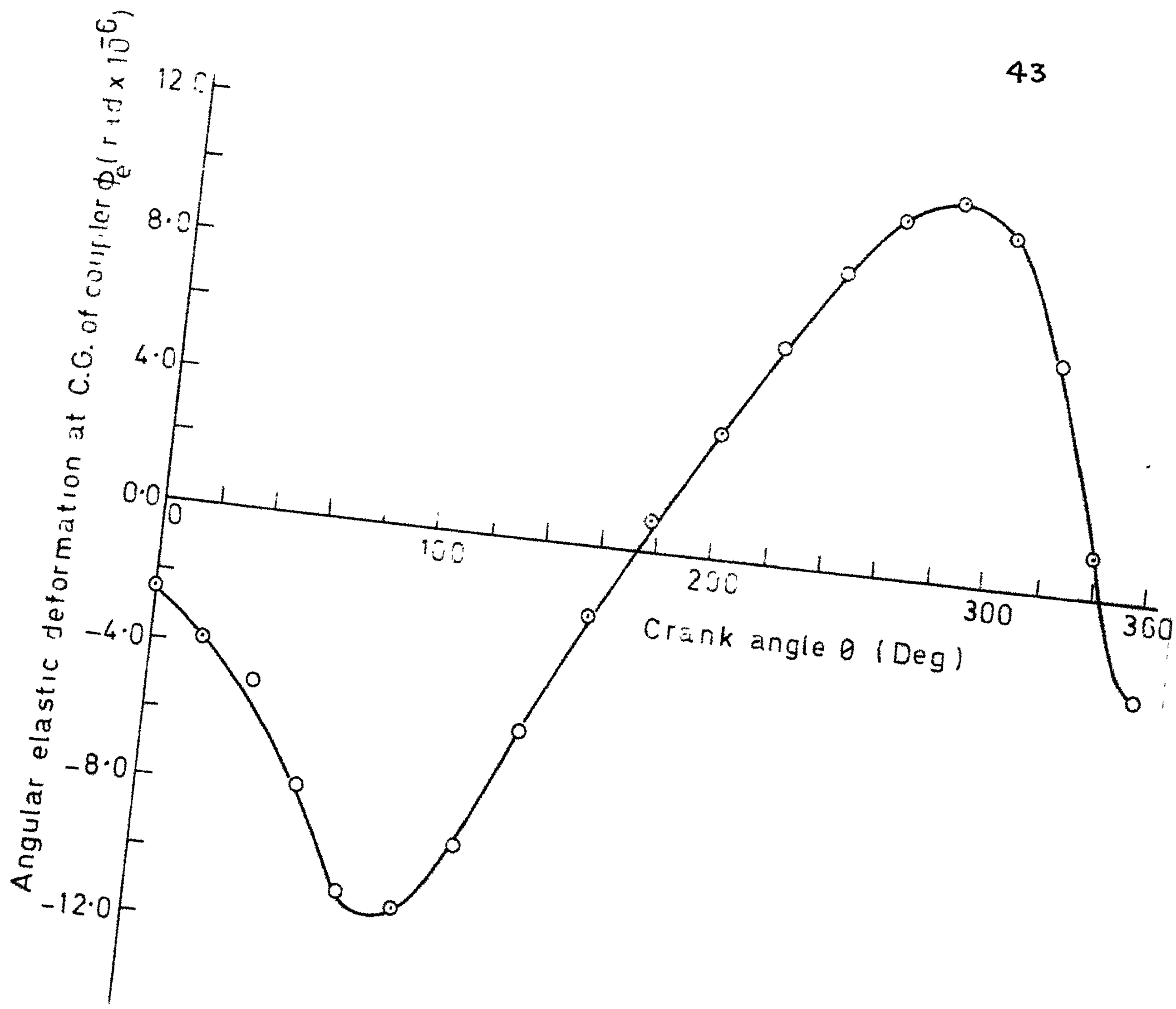

Fig.12 Variation of angular elastic deformation at centre of gravity of coupler with respect to crank angle

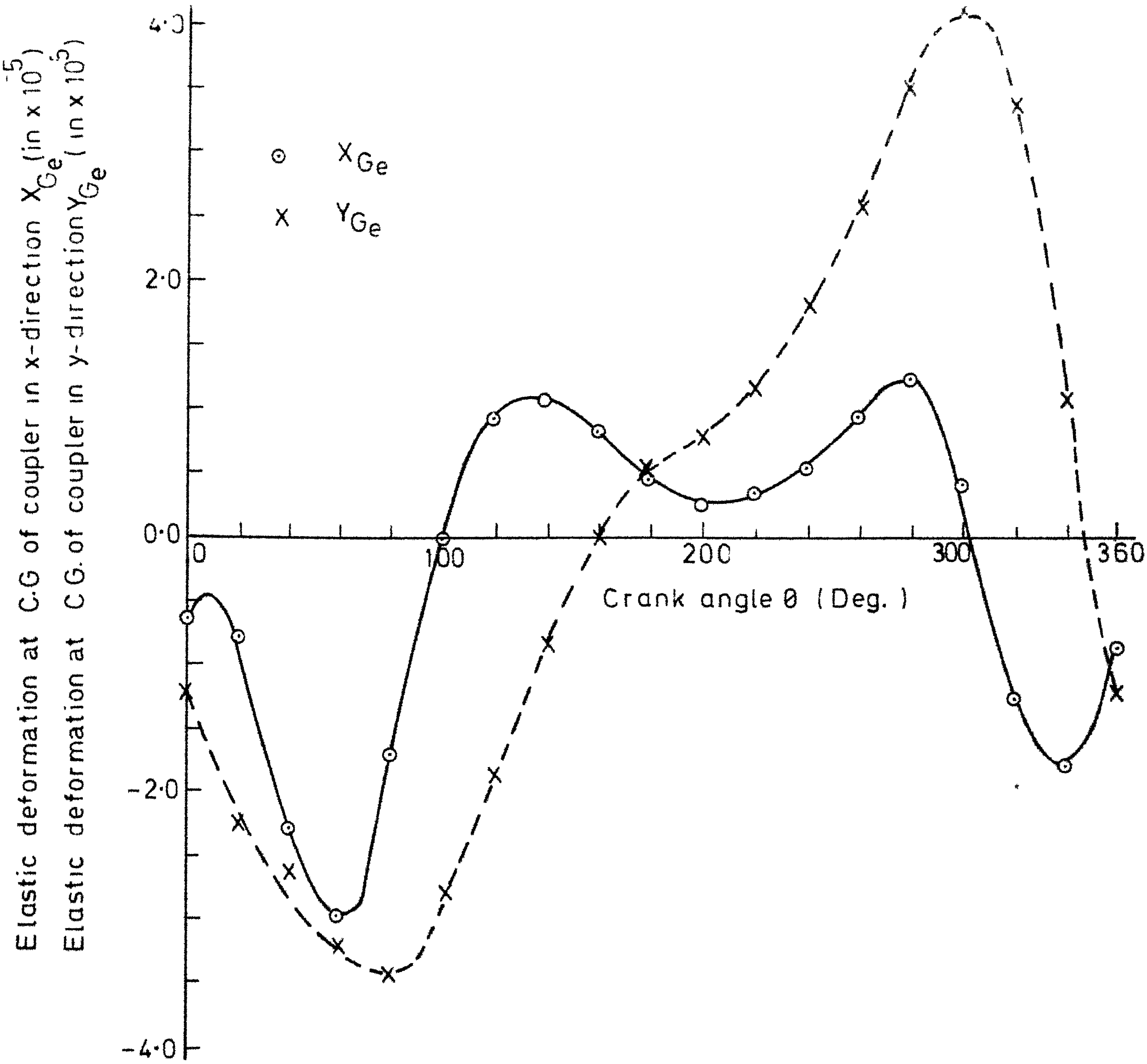

Fig 13 Variation of elastic deformations at centre of gravity of coupler in x and y-directions with respect to crank angle

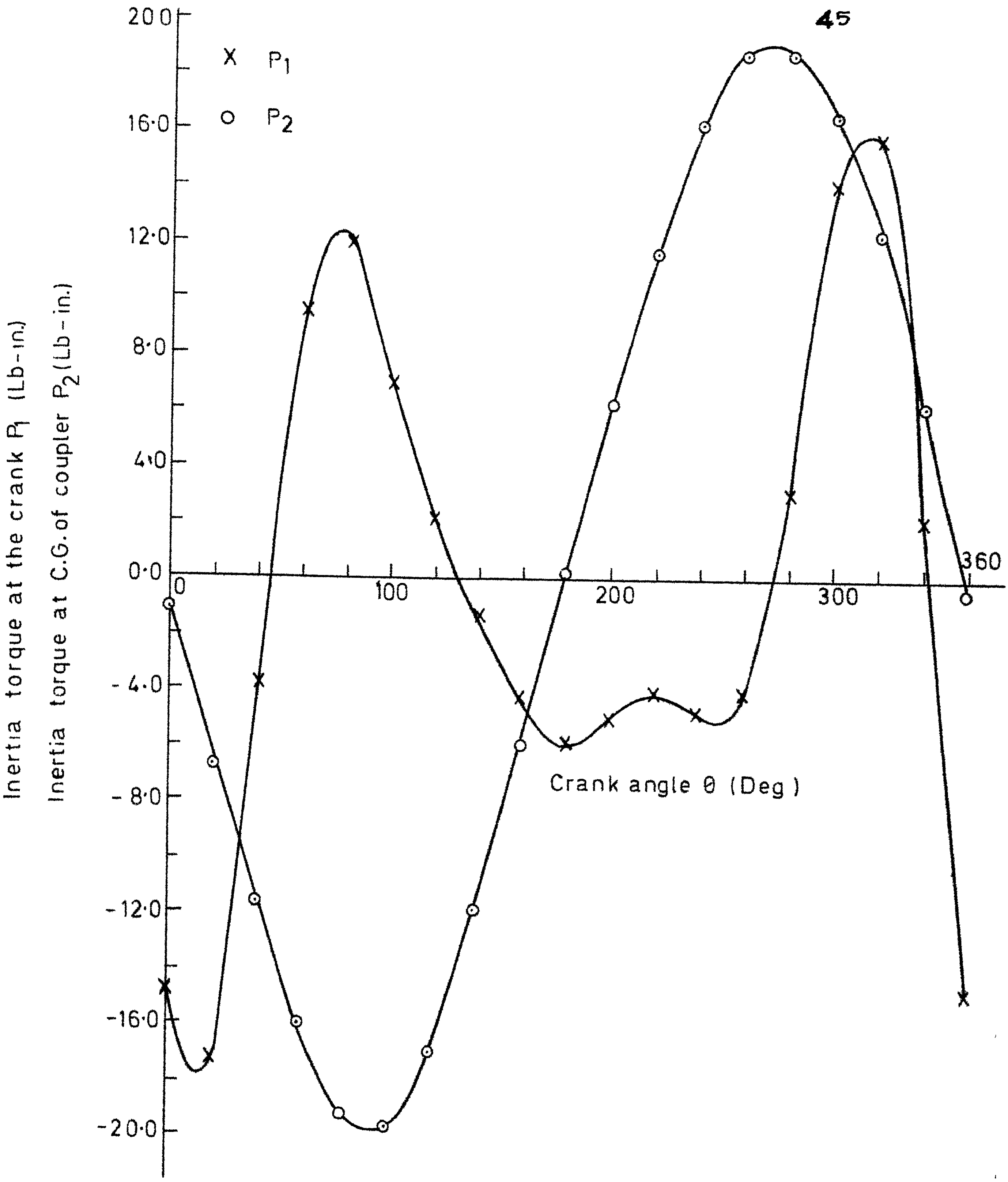

Fig. 14 Variation of inertia torques at the crank and the centre of gravity of the coupler with respect to crank angle

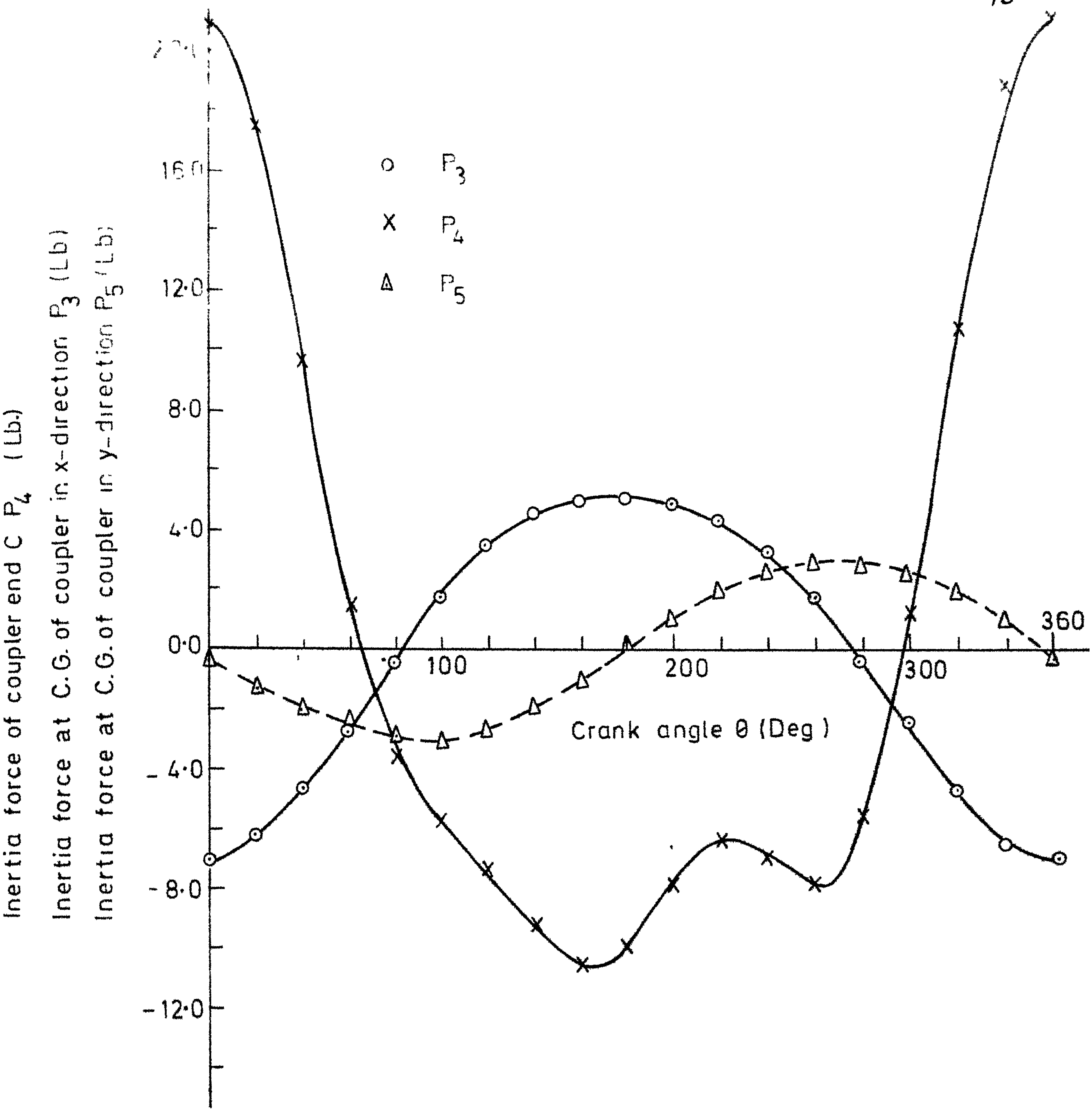

Fig. 15 Variation of Inertia forces at coupler end C and at centre of gravity of coupler in x and y directions with respect to crank angle

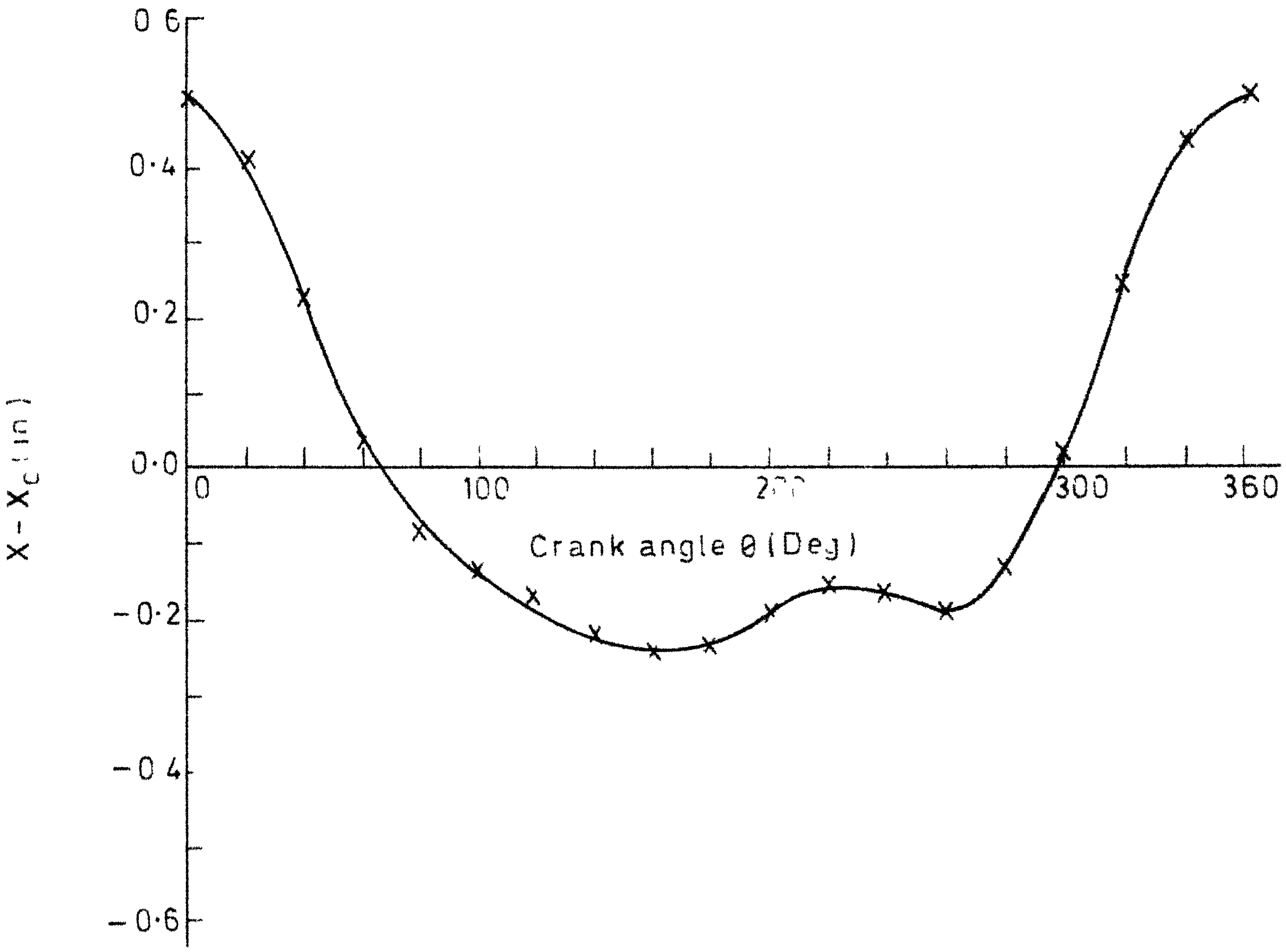

Fig.16 Variation of $(X-X_c)$ with respect to crank angle θ .

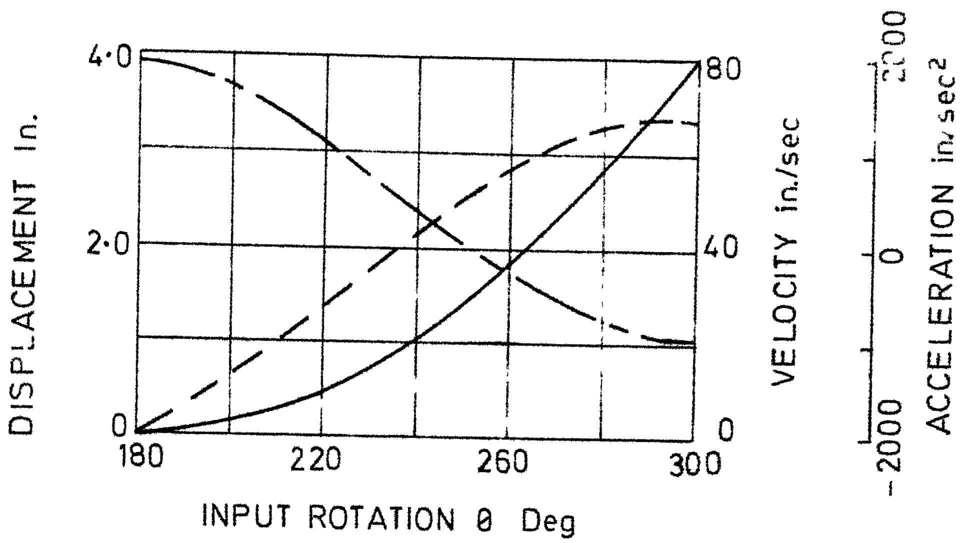

Fig.17 Motion characteristics for solution mechanism of Davidson's model

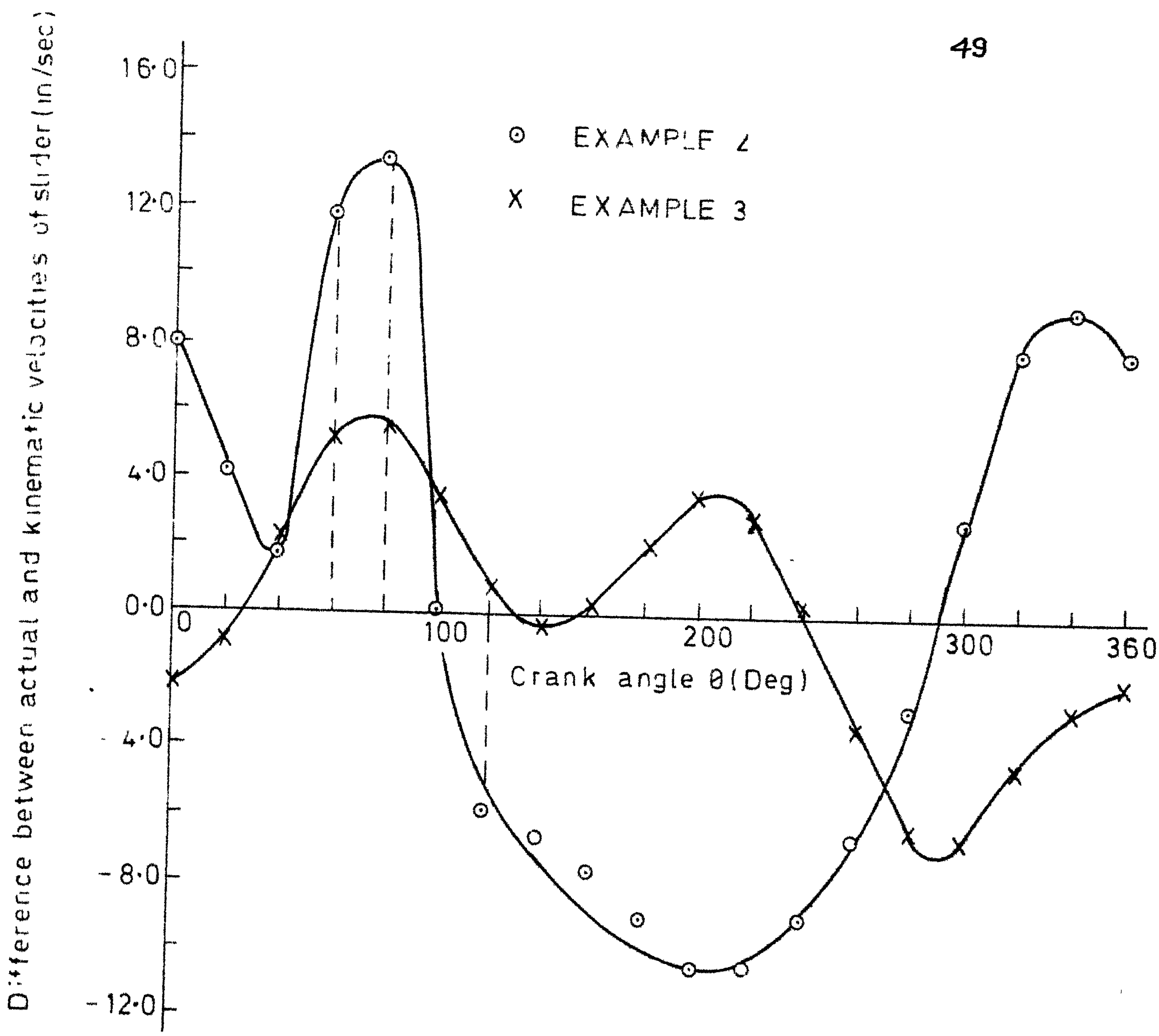

Fig.18 Variation of difference between actual and "Rigid" velocities of slider with respect to crank angle (Examples 2and 3)

of inertia about the transverse axis passing through its centre of gravity is taken as 0.153 lb-in-sec.^{2}. The modulus of elasticity of the material of crank and coupler is taken as 30×10^6 lb/in^2. The constant a_2 of the motor torque is taken as 1.8. [26]

3.1.1 Results and Discussion For Example Problem 1

With the above input data KEDA of the mechanism was carried out. The results obtained are represented graphically as shown in Figures 5 to 16.

Figure 5 represents the fluctuation of angular velocity $\dot{\theta}$ of the crank. The desired average speed of the crank is 21.0 rad/sec while the actual average speed of the crank is allowed to fall in the region represented in between the lines 1 and 2 (FIG.5), permitting an error of 0.5%.

Figure 6 represents the variation of slider displacement X and "rigid" displacement $\overline{X}_C$ of coupler - end C with respect to crank rotation θ. In the absence of spring and with rigid links, curve 2 would have represented the variation of slider displacement X. Therefore, the vertical ordinate between curves 1 and 2 at any value of θ represents the combined effect of inclusion of the spring and inherent elasticity of links of the mechanism on slider displacement X at that position of the crank. It may be noted that the characteristics of the mechanism are calculated taking into account the variation of crank speed $\dot{\theta}$ due to the changing inertia of the system with crank rotation.

Figure 7 represents variation of angular velocity of the coupler $\dot{\phi}$, while Figure 8 represents the variation of slider velocity $\dot{X}$ and x and y - directional velocities of the centre of gravity of the coupler with respect to θ. As may be expected, the slider velocity is not zero at crank angles 0°, 180° and 360°.

Figure 9 represents the variation of angular acceleration of crank and coupler with respect to θ. Refering to Figure 5, since the crank - speed decreases from $\theta = 0°$ to $\theta = 48°$, $\ddot{\theta}$ is negative in this range. Between $\theta = 48°$ and $\theta = 130°$, $\ddot{\theta}$ is positive as $\dot{\theta}$ continuously increases in this range. Similarly between $\theta = 130°$ and $\theta = 275°$, $\ddot{\theta}$ is negative and from $\theta = 275°$ to $\theta = 340°$ $\ddot{\theta}$ is positive. From 340° to 360° of crank rotation, $\ddot{\theta}$ is again negative.

Variations of linear accelerations of centre of gravity of the coupler in x and y-directions and slider acceleration with θ are shown in Figure 10.

The variation of elastic deformation X_{C_e} at coupler end C with θ is represented in Figure 11, while Figure 12 and 13 represent how the remaining element deflections vary with θ. The elastic deformation X_{C_e} (FIG. 11) when added to 'rigid' displacement $\overline{X}_C$ (FIG. 6) will yield the actual displacement X_C of the coupler end C. The variation of the length of spring $(X - X_C)$ with respect to θ is shown in Figure 16, while the variations of inertia forces with θ are plotted in Figure 14 and 15.

Refering to Figures 9, 10, 11, 14, 15 and 16 it may be noticed that there is a change in the nature of the curves of $\ddot{\theta}$, $\ddot{X}$, X_{C_e}, P_1, P_4 and $(X - X_C)$ in between $\theta = 160°$ and $\theta = 260°$. This may be attributed to the nature of variation of X_{C_e} and hence $(X - X_C)$ with respect to θ. The elastic deformation X_{C_e} varies between 160° and 260° of crank rotation in such a way that $(X - X_C)$ is affected in this range of θ as shown in Figure 16. The variation of $(X - X_C)$ directly affects inertia force P_4 (equation 2.30) and slider acceleration $\ddot{X}$ (equation 2.1), which in turn affect the input crank acceleration $\ddot{\theta}$ and inertia torque P_1.

The variations of slider displacement, velocity and acceleration with input angle θ as obtained by Davidson [5] for a schematically identical mechanism, are shown in Figure 17. Comparing his results with those obtained in the present analysis it was found that as may be expected, they do not tally. The difference in the results of slider acceleration is much more pronounced in comparision to the differences in displacement and velocity results. As per Davidson's results the slider moves 4.0 in. as the crank rotates from 180° to 300° while the present analysis gives this distance as 3.99 in. Davidson predicts the slider velocity at $\theta = 180°$ as 0.0 in./sec., at $\theta = 240°$ as 42.3 in/sec and at $\theta = 300°$ as 66.0 in/sec. The corresponding values obtained by the present analysis are 2.269, 39.352 and 67.8 in/sec respectively.

Since Davidson did not consider the effect of inherent elasticity, the fluctuation in driving link speed and the mass of crank and coupler (which have been accounted in the present analysis) the present results for $\ddot{X}$ are much different from one obtained by him. He obtains values of $\ddot{X}$ at 180°, 250° and 300° of crank rotation as 1900, 0.0 and - 900 in/sec^2 approximately as against 964.4, 740.0 and - 300.0 in/sec^2 obtained in the present analysis.

3.2 Examples 2 and 3

The mechanisms studied in examples 2 and 3 are schematically identical to those synthesized by Liniecki [26]. In Liniecki's first model the radius of crank, length of coupler and the amount of offset are taken to be 5.889 in., 19.164 in. and 0.688 in. while in the second model these values are taken as 5.933 in., 26.961 in. and 2.59 in. respectively. In both the models following values have been adopted by Liniecki :

Mass of the slider	=	0.52 lbf in^{-1} sec^2
Moment of inertia of crank about its axis of rotation	=	24.0 lbf in sec^2
Driving Torque Constant a_2	=	1.8
Average speed of crank Ω	=	26.2 rad/sec

Since Liniecki has considered rigidly attached slider, the stiffness of the spring in the present case is taken to be very high (52000.0 lbf/in.). Liniecki has not considered mass of

the coupler separately (he considers equivalent moment of inertia of rotating masses and equivalent reciprocating mass in the mechanism) and has assumed "rigid" links. The following additional information is assumed for both the models examined here.

Area of cross-section of crank	=	2.0 in.^{2}
Area of cross-section of coupler	=	1.5 in.^{2}
Modulus of Elasticity E	=	$30 * 10^6$ lbf/in.^{2}
Moment of inertia of coupler about the transverse axis passing through its centre of gravity.	=	5.1 lbf in. sec.^{2}

The mass of the coupler is assumed to be half that of the slider.

3.2.1 Results and Discussion of Example Problems 2 and 3

The KEDA of the mechanisms of examples 2 and 3 was performed. The difference between the actual velocity of slider and the desired velocity [26] was obtained for every 20° of crank rotation. Finally this quantity was plotted in Figure 18 against crank rotation θ.

Liniecki synthesizes the first mechanism by making the error between the actual and the desired velocities of the slider zero at 60°, 80° and 120° of crank rotation. The second mechanism is synthesized by making this error zero at 60°, 160° and 240° of crank rotation. But the results of KEDA of both the mechanisms predict that the velocities at these precision points will not be zero (Fig. 18) if elasticity is also taken into account. This error at precision points varies from 3% to 10%.

CHAPTER IV

CONCLUSIONS AND SCOPE FOR FURTHER WORK

The importance of KEDA of mechanisms has long been recognized by the researchers. As it is clear from the example problems worked out, this analysis is essential for the mechanisms where accuracy is main objective, since the elastic deformations may cause appreciable error in mechanism characteristics.

However, in the present dissertation the effects of clearances and tolerances, damping characteristics and friction in joints have not been taken into account. An analysis considering all these effects together would be an ideal one and will predict the actual characteristics of the mechanism much more closely. Efforts in future may be directed to formulate a mathematical model of mechanisms considering all these effects and to device suitable solution techniques.

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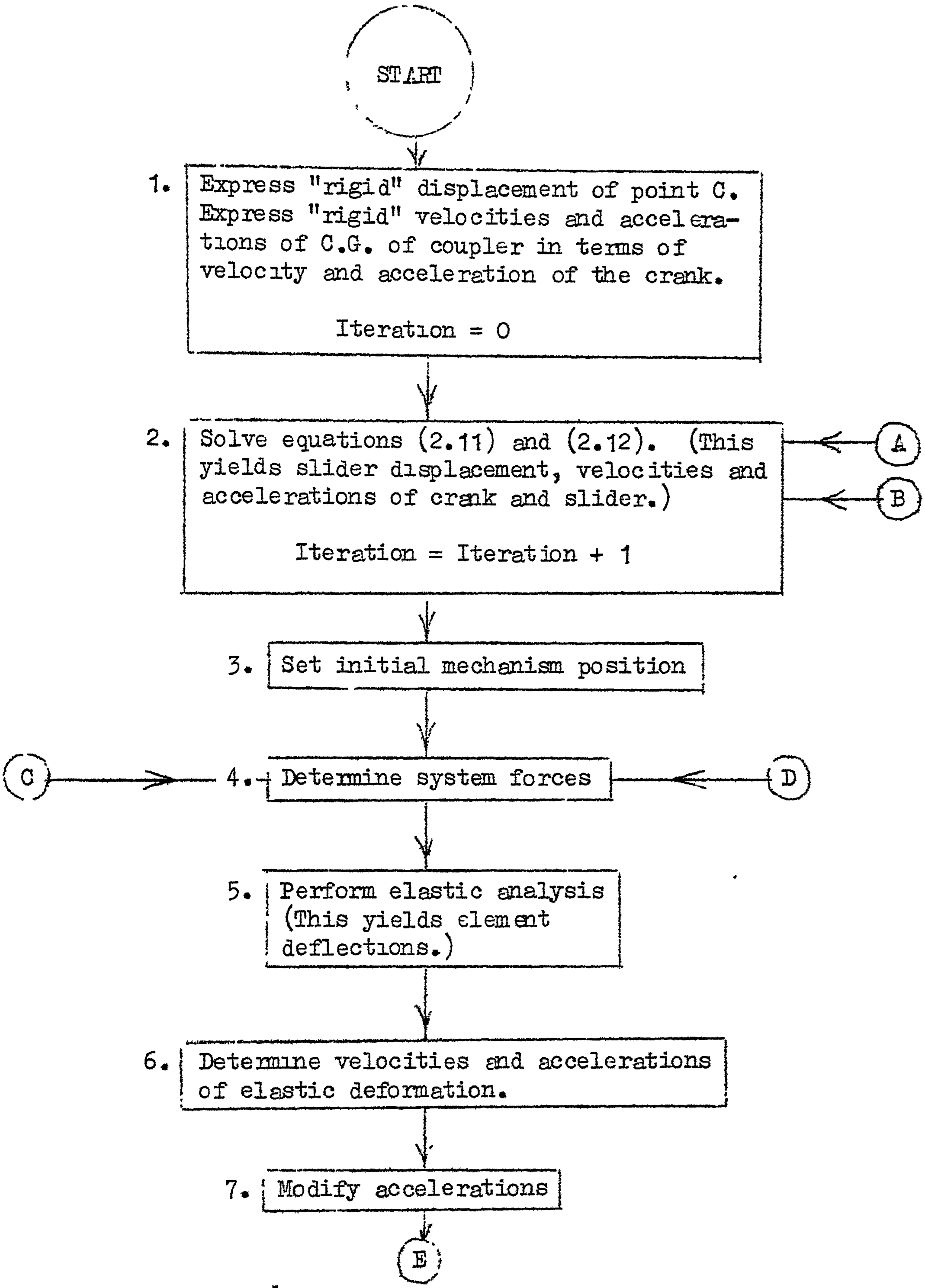
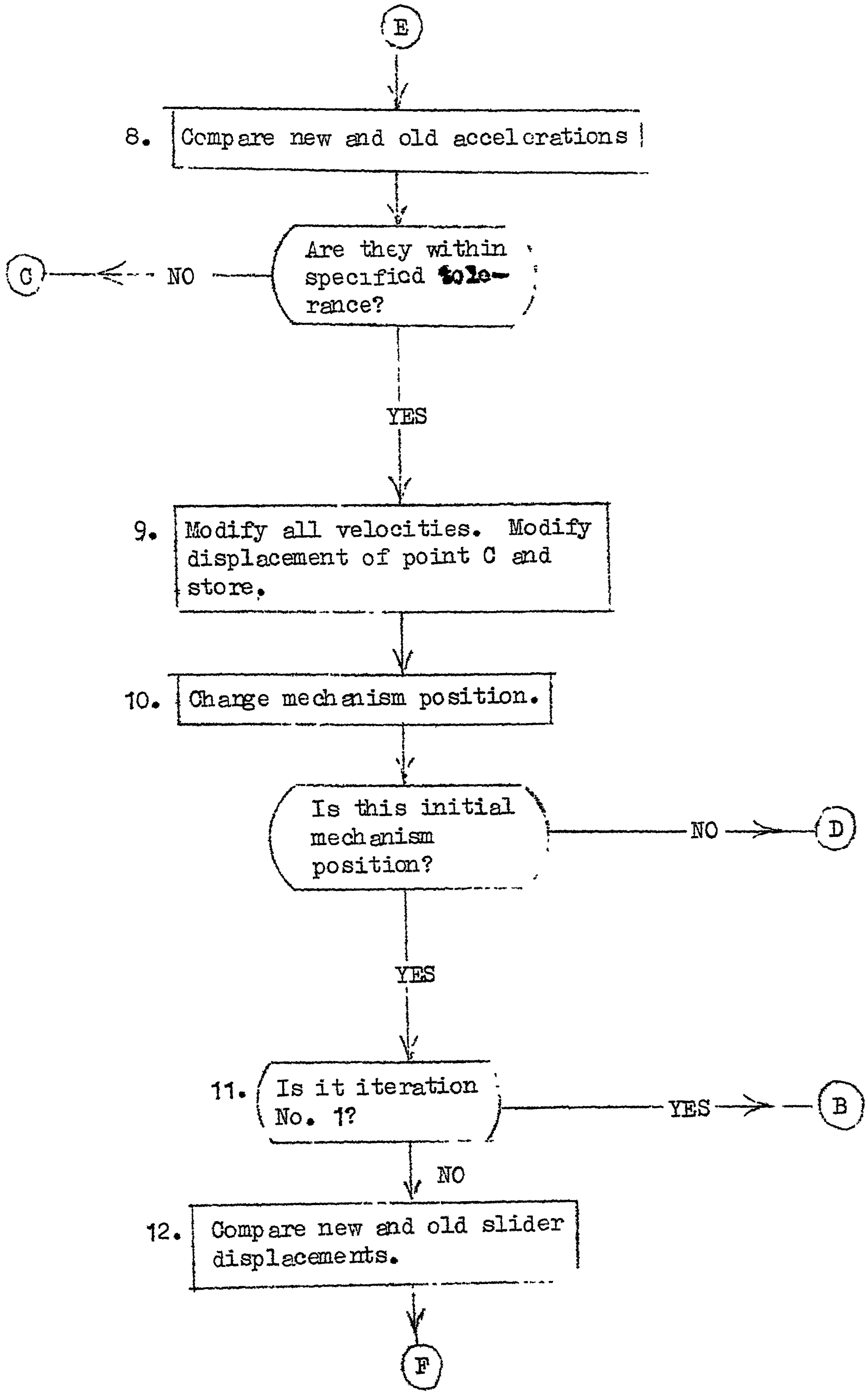
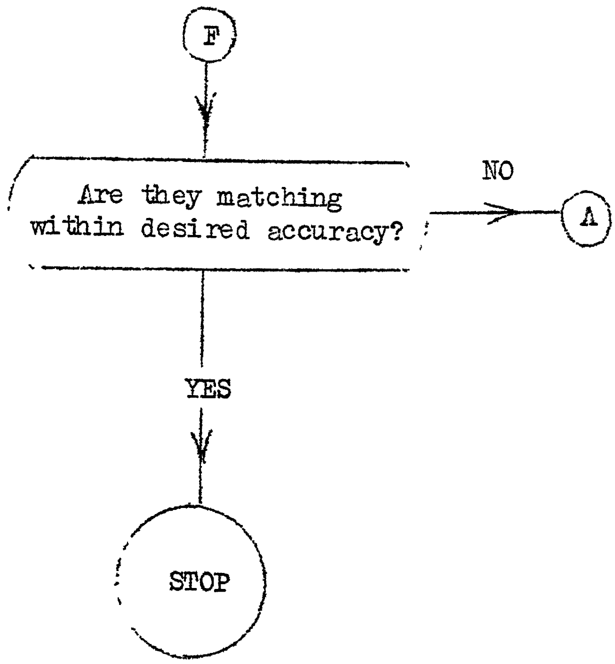
APPENDIX A

FLOW - CHART AND COMPUTER PROGRAMME LISTING

A - 1 FLOW CHART




E
8. Compare new and old accelerations
Are they within specified tole-rance?
NO
C
YES
9. Modify all velocities. Modify displacement of point C and store.
10. Change mechanism position.
Is this initial mechanism position?
NO
D
YES
11. Is it iteration No. 1?
YES
B
NO
12. Compare new and old slider displacements.
F


F
Are they matching
within desired accuracy?
NO
A
YES
STOP

```
*IBJOB
*IBFTC MAIN
                          ********************************
                   A-2    * COMPUTER PROGRAMME LISTING   *
                          ********************************

C                         ****************
C                         MAIN PROGRAMME
C                         ****************
C      ONE DIMENSIONAL ARRAY ABC STORED ALL THE INPUT DATA
       COMMON/PUSHPA/ABC
       DIMENSION   ABC( 5)
       NK=3
       DO  1   I=1,NK
       READ100,(ABC(J),J=1,24)
  100  FORMAT(8F10.5)
       ABC(19)=30000000.0
       IF(I.EQ.1)GO  TO  12
       ABC(13)=52.0  .
       ABC(14)=1000.0
  12   CONTINUE
       PRINT111,I
  111  FORMAT(50X,*PROBLEM  NO.  =*,I2/5 ,2 (1H-)/)
       PRINT101,(ABC(J),J=1,24)
  101  FORMAT(10X,*ABC*/4(1X,6F2 ).5/))
       CALL  ANALYS
  10   CONTINUE
       STOP
       END
```

```
*IBFTC ANALYS

      SUBROUTINE ANALYS

C     ****************************************************************
C     TOPIC---KINETO-ELASTODYNAMIC ANALYSIS OF SLIDER-CRANK MECHANISM
C           WITH FLEXIBLY ATTACHED SLIDER
C     ****************************************************************
C     FOLLOWING NOTATION
C     FOLLOWING NOTATIONS HAVE BEEN USED
C     ----------------------------------------------------------------
C     R---RADIOUS OF THE CRANK
C     AL-----LENGTH OF THE COUPLER
C     A---AMOUNT OF OFFSET
C     OMEGA---AVERAGE ANGULAR VELOCITY OF THE CRANK
C     OM1---MASS OF THE SLIDER
C     OM2---MASS OF THE COUPLER
C     S---STIFFNESS OF THE SPRING
C     E---MODULUS OF ELASTICITY OF THE MATERIAL OF CRANK * COUPLER
C     WN---SQUARE ROOT OF (S/OM1)
C     UI---MOMENT OF INERTIA OF CRANK ABOUT ITS AXIS OF ROTATION
C     GI---MOMENT OF INERTIA OF COUPLER ABOUT ITS C.G.
C     AO---CROSS-SECTIONAL AREA OF CRANK
C     AG---AREA OF CROSS-SECTION OF COUPLER
C     I1---SECTION MODULUS OF THE CRANK
C     I2---SECTION MODULUS OF THE COUPLE
C     ANGVEL---ANGULAR VELOCITY OF THE CRANK
C      Y---SQUARE OF ANGVEL
C     T---SMALL INCREMENT IN THE INPUT CRANK ANGLE THETA
C     XDOT---VELOCITY OF THE SLIDER
C     XFDOT---VELOCITY OF THE SLIDER BASED ON AVERAGE CRANK SPEED AS USED
C           BY DAVIDSON (REF. 5)
C     XPRIME---DERIVATIVE OF SLIDER DISPLACEMENT WITH RESPECT TO CRANK ANGLE
C     XD---DISPLACEMENT OF SLIDER
C     XFINAL---SLIDER DISPLACEMENT BASED ON AVERAGE CRANK SPEED AS USED BY
C           DAVIDSON (REF.5)
C     XC---KINEMATIC DISPLACEMENT OF COUPLER END C
C     XCF---DISPLACEMENT OF C CONSIDERING ELASTIC EFFECTS
C     XGDOT---X-DIRECTIONAL VELOCITY OF C.G. OF THE COUPLER
C     YGDOT---Y-DIRECTIONAL VELOCITY OF C.G. OF THE COUPLER
C     PHYDOT---ANGULAR VELOCITY OF THE COUPLER
C     ALPHA---ANGULAR ACCELERATION OF TH  CRANK
```

```
C     BETA----ANGULAR ACCELERATION OF THE COUPLER
C     ACCX---SLIDER ACCELERATION
C     ACCXG---X-DIRECTIONAL ACCELERATION OF C.G. OF THE COUPLER
C     ACCYG---Y-DIRECTIONAL ACCELERATION OF C.G. OF THE COUPLER
C     Q1 TO Q5---INERTIA FORCES
C     XCDEL---LINEAR ELASTIC DEFORMATION AT POINT C OF THE COUPLER
C     PHYDEL---ANGULAR ELASTIC DEFORMATION AT C.G. OF THE COUPLER
C     XGDEL---X-DIRECTIONAL ELASTIC DEFORMATION AT C.G. OF COUPLER
C     YGDEL---Y-DIRECTIONAL ELASTIC DEFORMATION AT C.G. OF COUPLER
C     DPHDOT---RATE OF ANG. ELASTIC DEF. AT C.G. OF THE COUPLER
C     DXGDOT---RATE OF X-DIRECTIONAL ELASTIC DEF. AT C.G. OF COUPLER
C     DYGDOT---RATE OF Y-DIRECTIONAL ELASTIC DEF. AT C.G. OF COUPLER
C     DBETA---ACC. OF ANG. ELASTIC DEF. AT C.G. OF COUPLER
C     DACCXG---ACC. OF X-DIRECTIONAL ELASTIC DEF. AT C.G. OF COUPLER
C     DACCYG---ACC. OF Y-DIRECTIONAL ELASTIC DEF. AT C.G. OF COUPLER


      REAL  I1,I2
      COMMON/PUSHPA/ABC
      COMMON/SMITA/XCFP
      COMMON/TAJ/ZS
      COMMON/KHARE/R,AL,A,OI,GI,OM1,OM2,G,W1,A1,A2
      DIMENSION  ABC(25)
      DIMENSION  ZS(6)
      DIMENSION  IMITA(181)
      DIMENSION  XFOUT(181)
      DIMENSION  DZY4(3),ZY(3)
      DIMENSION  PHYDEL(181),XGDEL(181),YGDEL(181),PHYDOT(181)
      DIMENSION  XDOT(181),XD(181)
      DIMENSION  ZX(3),ZZ(182, ),DZX(3),DZY1(3),DZY2(3),DZY3(3)
      DIMENSION  ALPHA(181),BETA(181),XDOUBL(181),ACCX(181),Y(181)
      DIMENSION  ACCXG(181),XCDEL(181),Q1(181),Q2(181),Q3(181),Q4(181)
       DIMENSION  Q5(181),XC(181),XCF(181),ACCYG(181),ANGVEL(181)
      DIMENSION  DPHDOT(181),DXGDOT(181),DYGDOT(181),DBETA(181),DACCXG(
     1181),DACCYG(181),XGDOT(181),YGDOT(181),XCF1(181)
      CALL  FLUN(ZS )
      PAI=4.0*ATAN(1.0)
      T=PAI/90.0
      EPS5=5.0
      EPS6=1.0
      ZX(1)=ABC(1)
      ZX(2)=ABC(2)
      ZX(3)=ABC(3)
      A1=ABC(4)
      R=ABC(5)
      AL=ABC(6)
      A=ABC(7)
      A2=ABC(8)
```

```
      DI=ABC(9)
      GI=ABC(10)
      DM1=ABC(11)
      DM2=ABC(12)
      S=ABC(13)
      WN=ABC(14)
      AO=ABC(15)
      AG=ABC(16)
      I1=ABC(17)
      I2=ABC(18)
      E=ABC(19)
      OMEGA=ABC(20)
      EPS1=ABC(21)
      EPS2=ABC(22)
      EPS3=ABC(23)
      EPS4=ABC(24)
      IRE=1
      MM=0
 17   MM=MM+1
      MK=1
      NK=1
      DO  1  I=1,3
      ZZ(1,I)=ZX(I)
      NN=180
      NJ=10
      N=181
      X=0.0
1111  K=1
      XCFF=0.0
      DO  10  KJ=1,NJ
      XD(KJ)=0.0
      KK=KJ+1
      X1=X+T/2.0
      X2=X+T
      CALL  FUNC(X,ZX,DZX,K,XCFF)
      ALPHA(KJ)=DZX(2)/2.0
      XDOUBL(KJ)=DZX(3)
      Y(KJ)=ZX(2)
      ACCX(KJ)=ZX(3)*ALPHA(KJ)+ZX(1)*XDOUBL(KJ)
      DO  20  I=1,3
      DZY1(I)=DZX(I)
 20   ZY(I)=ZX(I)+0.5*T*DZX(I)
      CALL  FUNC(X1,ZY,DZX,K,XCFF)
      DO  25  I=1,3
      DZY2(I)=DZX(I)
 25   ZY(I)=ZX(I)+ .5*T*DZX(I)
      CALL  FUNC(X1,ZY,DZX,K,XCFF)
      DO  30  I=1,3
      DZY3(I)=DZX(I)
 30   ZY(I)=ZX(I)+T*DZX(I)
```

```
      DI=ABC(9)
      GI=ABC(10)
      DM1=ABC(11)
      DM2=ABC(12)
      S=ABC(13)
      WN=ABC(14)
      AO=ABC(15)
      AG=ABC(16)
      I1=ABC(17)
      I2=ABC(18)
      E=ABC(19)
      OMEGA=ABC(20)
      EPS1=ABC(21)
      EPS2=ABC(22)
      EPS3=ABC(23)
      EPS4=ABC(24)
      IRE=0
      MM=0
   17 MM=MM+1
      MK=1
      NK=1
      DO  1  I=1,3
    1 ZZ(1,I)=ZX(I)
      NN=180
      NJ=10
      N=181
      X=0.0
 1111 K=1
      XCFF=0.0
      DO  10  KJ=1,N
      XD(KJ)=0.0
      KK=KJ+1
      X1=X+T/2.0
      X2=X+T
      CALL  FUNC(X,ZX,DZX,K,XCFF)
      ALPHA(KJ)=DZX(2)/2.0
      XDOUBL(KJ)=DZX(3)
      Y(KJ)=ZX(2)
      ACCX(KJ)=ZX(3)*ALPHA(KJ)+ZX(2)*XDOUBL(KJ)
      DO  20  I=1,3
      DZY1(I)=DZX(I)
   20 ZY(I)=ZX(I)+0.5*T*DZX(I)
      CALL  FUNC(X1,ZY,DZX,K,XCFF)
      DO  25  I=1,3
      DZY2(I)=DZX(I)
   25 ZY(I)=ZX(I)+0.5*T*DZX(1)
      CALL  FUNC(X1,ZY,DZX,K,XCFF)
      DO  30  I=1,3
      DZY3(I)=DZX(I)
   30 ZY(I)=ZX(I)+T*DZX(I)
```

```
      CALL  FUNC(X2,ZY,DZX,K,COEF)
      DO  35  I=1,3
      DZY4(I)=DZX(I)
      ZX(I)=ZX(I)+T*(DZY1(I)+2.*DZY2(I)+2.*DZY3(I)+DZY4(I))/6.
35    ZZ(KK,I)=ZX(I)
      X=X+T
10    CONTINUE
834   CONTINUE
      HH1=ZZ(1,1)-ZZ(181,1)
      HH2=ZZ(1,2)-ZZ(181,2)
      HH3=ZZ(1,3)-ZZ(181,3)
      YAV=0.0
      DO  797  I=1,180
797   YAV=YAV+ZZ(I,2)
      YAV=YAV/180.0
      AV=OMEGA**2
      HH4=YAV-AV
      IF(ABS(HH1).LE.EPS1)GO  TO  250
      MK=2
      ZZ(1,1)=ZZ(1,1)-0.5*HH1
250   IF(ABS(HH2).LE.EPS6)GO  TO  252
      MK=2
      A1=A1+5.0*HH2/3.0
252   IF(ABS(HH3).LE.EPS3)GO  TO  254
      MK=2
      ZZ(1,3)=ZZ(1,3)-0.5*HH3
254   IF(ABS(HH4).LE.EPS5)GO  TO  257
      MK=2
      ZZ(1,2)=ZZ(1,2)-HH4
257   IF(MK.NE.2)GO  TC  255
      DO  795  I=1,3
795   ZX(I)=ZZ(1,I)
      IF(NK.EQ.3)GO  TC  836
       GO  TO  17
255   CONTINUE
      PRINT99,MM
99    FORMAT(50X,*TOTAL ITERATIONS TAKEN=*,I3/50X,22(1H-))
      PRINT750,A1
750   FORMAT(20X,*A1=*,F15.3)
      PRINT100,(ZZ(K,1),K=1,181,10)
100   FORMAT(50X,*SLIDER DISPLACEMENT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT101,(ZZ(K,2),K=1,181,10)
101   FORMAT(50X,*VALUE  OF  Y*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT102,(ZZ(K,3),K=1,181,10)
102   FORMAT(50X,*VALUE  OF  XPRIM*//6(10X,F15.8,5(5X,F15.8)/))
      DO  200  I=1,181
200   XDOT(I)=ZZ(I,3)*SQRT(ZZ(I,2))
      PRINT103,(XDOT(I),I=1,181,10)
103   FORMAT(50X,*VALUE  OF  XDOT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT256
```

```
256   FORMAT(50X,*CONVERGENCE IS SATISFIED*)
633   CONTINUE
      I=0
      X=0.0
      P3=R**2/(4.0*AL)
      P4=A**2/(2.0*AL)
      P7=R*A/AL
      P11=R/AL
3     I=I+1
      IMITA(I)=0
      II=I+1
      IJ=I+2
      XX=2.0*X
      ANGVEL(I)=SQRT(Y(I))
      P5=WN**2/(WN**2-Y(I))
      P6=WN**2/(WN**2-4.0*Y(I))
      P8=AL*(1.0-(A/AL+R*SIN(X)/AL)**2)**0.5
      P9=P8**2
      P10=(A+R*SIN(X))*(R*COS(X))**2
      P12=R*SIN(X)+0.5*P11*(0.5*R*SIN(XX)+A*COS(X))
      P13=R*COS(X)+0.5*P11*(R*COS(XX)-A*SIN(X))
      Z=ARSIN(A/AL+R*SIN(X)/AL)
      P14=E*A0
      P15=E*I1
      P16=E*AG
      P17=SIN(X)+TAN(Z)*COS(X)
      P18=COS(X)-TAN(Z)*SIN(X)
      P19=COS(X)-.5*TAN(Z)*SIN(X)
      P20=SIN(X)+.5*TAN(Z)*COS(X)
      P21=TAN(Z)/COS(Z)
      P22=1.0/COS(Z)**2
      P23=P18**2
      P24=P17**2
      BB1=0.5*AL**2*P17/P15
      BB21=-SIN(X)*P18/(P14*COS(Z))
      BB22=AL*AL*COS(X)*P17/(2.*P15*COS(Z))
      BB23=P21/P16
      BB2=BB21+BB22+BB23
      BB31=AL*P19*P18/P14+.5*AL*P22/P16
      BB32=AL*AL*AL*P17*P20/(6.0*P15)
      BB3=-BB31-BB32
      BB4=AL*P23/P14+AL*P22/P16+AL**3*P24/(3.0*P15)
      BB51=AL*P18*SIN(X)/(2.*P14)
      BB52=AL*AL*AL*P17*COS(X)/(6.*P15)
      BB5=BB52-BB51
      XC(I)=R*COS(X)+P8
      ALP1=AL/(E*A0)
      ALP2=AL**3/(3.0*E*I1)
      ALP3=AL**2/(2.0*E*I1)
      ALP4=AL/(E*I1)
```

```
      ALP5=AL*P22/(E*AG)
      ALP6=SIN(Z)*P22/(E*AG)
      ALP7=-AL*P22/(2.0*E*AG)
      ALP8=AL/(12.0*E*I2)+TAN(Z)**2/(AL*E*AG)
      ALP9=-0.5*ALP6
      ALP10=AL*(P22+COS(Z)**2)/(4.0*E*AG)+AL**3*SIN(Z)**2/(48.0*E*I2)
      ALP11=AL**3*SIN(2.0*Z)/(96.0*E*I2)-AL*SIN(2.0*Z)/(8.0*E*AG)
      ALP12=AL*SIN(Z)**2/(4.0*E*AG)+AL**3*COS(Z)**2/(48.0*E*I2)
      B1=-SIN(X)/(AL*COS(Z))
      B2=-P19
      B3=P18
      B4=-0.5*SIN(X)
      B5=COS(X)/(AL*COS(Z))
      B6=-P20
      B7=P17
      B8=0.5*COS(X)
      BB6=ALP3*B5
      BB7=ALP1*B1**2+ALP2*B5**2+ALP5
      BB8=ALP1*B1*B2+ALP2*B5*B6+ALP9
      BB9=ALP1*B1*B3+ALP2*B5*B7+ALP6
      BB10=ALP1*B1*B4+ALP2*B5*B8
      BB11=ALP3*B6
      BB12=ALP1*B1*B2+ALP2*B5*B6+ALP9
      BB13=ALP1*B1**2+ALP2*B6**2+ALP10
      BB14=ALP1*B2*B3+ALP2*B6*B7+ALP7
      BB15=ALP1*B2*B4+ALP2*B6*B8+ALP11
      BB16=ALP3*B8
      BB17=ALP1*B1*B4+ALP2*B5*B8
      BB18=ALP1*B2*B4+ALP2*B6*B8+ALP11
      BB19=ALP1*B3*B4+ALP2*B7*B8
      BB20=ALP1*B4**2+ALP2*B8**2+ALP12
      IF(IRE.GE.1)GO  TO  962
      XCF(I)=XC(I)
962   CONTINUE
      XFDOT(I)=-ANGVEL(I)*(R*P5*SIN(X)+P1*P5*COS(X)+2.*P3*P6*SIN(XX))
      ACCXG(I)=-P12*ALPHA(I)-P13*Y(I)
      ACCYG(I)=0.5*R*(COS(X)*ALPHA(I)-SIN(X)*Y(I))
      BETA(I)=(R*COS(X)*ALPHA(I)+(P13/P9-R*SIN(X))*Y(I))/P8
      Q1(I)=OI*ALPHA(I)
      Q2(I)=GI*BETA(I)
      Q3(I)=OM2*ACCXG(I)
      Q4(I)=S*(ZZ(I,1)-XCF(I))
      Q5(I)=OM2*ACCYG(I)
      XCDEL(I)=BB1*Q1(I)+BB2*Q2(I)+BB3*Q3(I)+BB4*Q4(I)+BB5*Q5(I)
      XCF(I)=XC(I)+XCDEL(I)
555   CONTINUE
      IMITA(I)=IMITA(I)+1
      PHYDEL(I)=BB6*Q1(I)+BB7*Q2(I)+BB8*Q3(I)+BB9*Q4(I)+BB10*Q5(I)
      XGDEL(I)=BB11*Q1(I)+BB12*Q2(I)+BB13*Q3(I)+BB14*Q4(I)+BB15*Q5(I)
      YGDEL(I)=BB16*Q1(I)+BB17*Q2(I)+BB18*Q3(I)+BB19*Q4(I)+BB20*Q5(I)
```

```
      IF(I.EQ.N)GO  TO  551
      HH1=(PHYDEL(II)-PHYDEL(I))/T
      HH2=(XGDEL(II)-XGDEL(I))/T
      HH3=(YGDEL(II)-YGDEL(I))/T
      IF(I.EQ.NN)IJ=1
      HH4=(PHYDEL(IJ)-2.0*PHYDEL(II)+PHYDEL(I))/T**2
      HH5=(XGDEL(IJ)-2.0*XGDEL(II)+XGDEL(I))/T**2
      HH6=(YGDEL(IJ)-2.0*YGDEL(II)+YGDEL(I))/T**2
      DBETA(I)=HH4*Y(I)+HH1*ALPHA(I)
      DACCXG(I)=HH5*Y(I)+HH2*ALPHA(I)
      DACCYG(I)=HH6*Y(I)+HH3*ALPHA(I)
      GO  TO  552
551   CONTINUE
      DBETA(I)=DBETA(1)
      DACCXG(I)=DACCXG(1)
      DACCYG(I)=DACCYG(1)
552   CONTINUE
      BETA(I)=DBETA(I)+(R*COS(X)*ALPHA(I)+(P10/P9-R*SIN(X))*Y(I))/P8
      ACCXG(I)=-P12*ALPHA(I)-P13*Y(I)+DACCXG(I)
      ACCYG(I)=DACCYG(I)+0.5*R*(COS(X)*ALPHA(I)-SIN(X)*Y(I))
      Q1(I)=OI*ALPHA(I)
      Q2(I)=GI*BETA(I)
      Q3(I)=OM2*ACCXG(I)
      Q4(I)=S*(ZZ(I,1)-XCF(I))
      Q5(I)=OM2*ACCYG(I)
      XCDEL(I)=BB1*Q1(I)+BB2*Q2(I)+BB3*Q3(I)+BB4*Q4(I)+BB5*Q5(I)
      XCF1(I)=XC(I)+XCDEL(I)
      HHK=(XCF1(I)-XCF(I))/XCF(I)
      XCF(I)=XCF1(I)
      IF(ABS(HHK).LT.EPS4)GO  TO  554
      GO  TO  555
554   CONTINUE
      IF(I.EQ.N)GO  TO  556
      DPHDOT(I)=HH1*ANGVEL(I)
      DXGDOT(I)=HH2*ANGVEL(I)
      DYGDOT(I)=HH3*ANGVEL(I)
      GO  TO  557
556   CONTINUE
      DPHDOT(I)=DPHDOT(1)
      DXGDOT(I)=DXGDOT(1)
      DYGDOT(I)=DYGDOT(1)
557   CONTINUE
      PHYDOT(I)=DPHDOT(I)+P11*COS(X)*ANGVEL(I)/COS(Z)
      YGDOT(I)=DYGDOT(I)+0.5*R*COS(X)*ANGVEL(I)
      XGDOT(I)=DXGDOT(I)-ANGVEL(I)*(R*SIN(X)+0.5*P11*(0.5*R*SIN(XX)+A*
     1COS(X)))
      X=X+T
      IF(I.LT.N) GO TO 3
      PRINT688,(IMITA(I),I=1,181,10)
688   FORMAT(10X,*NO OF ITTRATIONS BEFORE CONVERGENCE*/1X,20I6)
```

```
      PRINT201,(XDOUBL(I),I=1,181,10)
  201 FORMAT(50X,*VALUE  OF  XDOUBL*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT202,(ANGVEL(I),I=1,181,10)
  202 FORMAT(/50X,*VALUE OF ANGVEL     *//6(10X,F15.8,5(5X,F15.8)/))
      PRINT203,(ACCX(I),I=1,181,10)
  203 FORMAT(50X,*SLIDER  ACC*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT204,(XFDOT(I),I=1,181,10)
  204 FORMAT(/50X,*VALUE OF XFDOT *//6(10X,F15.8,5(5X,F15.8)/))
      PRINT205,(ALPHA(I),I=1,181,10)
  205 FORMAT(/50X,*ANG  ACC*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT206,(BETA(I),I=1,181,10)
  206 FORMAT(/50X,*VALUE OF BETA *//6(10X,F15.8,5(5X,F15.8)/))
      PRINT207,(ACCXG(I),I=1,181,10)
  207 FORMAT(/50X,*VALUE OF ACCXG*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT2070,(ACCYG(I),I=1,181,10)
 2070 FORMAT(/50X,* ACCYG        *//6(10X,F15.8,5(5X,F15.8)/))
      PRINT212,(Q1(I),I=1,181,10)
  212 FORMAT(/50X,*VALUE  OF  Q1*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT213,(Q2(I),I=1,181,10)
  213 FORMAT(/50X,*VALUE  OF  Q2*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT214,(Q3(I),I=1,181,10)
  214 FORMAT(/50X,*VALUE  OF  Q3*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT215,(Q4(I),I=1,181,10)
  215 FORMAT(/50X,*VALUE  OF  Q4*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT2152,(Q5(I),I=1,181,10)
 2152 FORMAT(50X,*VALUE  OF  Q5*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT216,(XCDEL(I),I=1,181,10)
  216 FORMAT(/50X,* VALUE OF XCDEL      *//6(10X,F15.8,5(5X,F15.8)/))
      PRINT217,(XC(I),I=1,181,10)
  217 FORMAT(/50X,*VALUE  OF  XC*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT219,(XCF(I),I=1,181,10)
  219 FORMAT(/50X,*FINAL  XCF*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT220,(PHYDEL(I),I=1,181,10)
  220 FORMAT(50X,*PHYDEL*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT221,(XGDEL(I),I=1,181,10)
  221 FORMAT(50X,*XGDEL*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT222,(YGDEL(I),I=1,181,10)
  222 FORMAT(50X,*YGDEL*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT223,(PHYDOT(I),I=1,181,10)
  223 FORMAT(50X,*PHYDOT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT224,(XGDOT(I),I=1,181,10)
  224 FORMAT(50X,*XGDOT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT225,(YGDOT(I),I=1,181,10)
  225 FORMAT(50X,*YGDOT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT226,(DPHDOT(I),I=1,181,10)
  226 FORMAT(50X,*DPHDOT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT227,(DXGDOT(I),I=1,181,10)
  227 FORMAT(50X,*DXGDOT*//6(10X,F15.8,5(5X,F15.8)/))
      PRINT228,(DYGDOT(I),I=1,181,10)
  228 FORMAT(50X,*DYGDOT*//6(10X,F15.8,5(5X,F15.8)/))
```

```
      PRINT229,(DBETA(I),I=1,[illegible])
229   FORMAT(50X,*DBETA*//6([illegible],F15.[illegible],5([illegible]X,F15.[illegible])/))
      PRINT230,(DACCXG(I),I=1,1[illegible])
230   FORMAT(50X,*DACCXG*//6([illegible],F15.[illegible],5(5[illegible],F15.[illegible])/))
      PRINT231,(DACCYG(I),I=[illegible])
231   FORMAT(50X,*DACCYG*//6([illegible],F15.[illegible],5(5X,F15.[illegible])/))
      DO  837  I=1,181
      HK=(ZZ(I,1)-XD(I))/ZZ(I,1)
      IF(ABS(HK).GE.0.0[illegible])GO  TO  [illegible]
837   CONTINUE
      PRINT839
839   FORMAT(50X,*PROBLEM  HAS  CONVERGED*/50X,[illegible](1H*))
      RETURN
838   DO  840  I=1,181
840   XD(I)=ZZ(I,1)
      IRE=IRE+1
      PRINT961,IRE
961   FORMAT(/50X,*REFIT  NO=*,I2/50X,1[illegible](1H*)//)
      MM=0
836   CONTINUE
      MM=MM+1
      K=2
      X=0.0
      DO  809  I=1,3
809   ZX(I)=ZZ(1,I)
      DO  810  JK=1,N
      KK=JK+1
      KKJ=KK+1
      X1=X+0.5*T
      X2=X+T
      XCFF=XCF(JK)
      XCFP=(XCDEL(KK)-XCDEL(JK))/T
      ZS(1)=PHYDOT(JK)
      ZS(2)=XGDOT(JK)
      ZS(3)=YGDOT(JK)
      ZS(4)=BETA(JK)
      ZS(5)=ACCXG(JK)
      ZS(6)=ACCYG(JK)
      CALL  FUNC(X,ZX,DZX,K,XCFF)
      ALPHA(JK)=DZX(2)/2.0
      XDOUBL(JK)=DZX(3)
      Y(JK)=ZX(2)
      ACCX(JK)=ZX(3)*ALPHA(JK)+ZX(2)*XDOUBL(JK)
      DO  820  I=1,3
      DZY1(I)=DZX(I)
820   ZY(I)=ZX(I)+0.5*T*DZX(I)
      XCFF=0.5*(XCF(JK)+XCF(KK))
      XCFP=0.5*(XCFP+(XCDEL(KKJ)-XCDEL(KK))/T)
      ZS(1)=0.5*(PHYDOT(JK)+PHYDOT(KK))
      ZS(2)=0.5*(XGDOT(JK)+XGDOT(KK))
```

```
      ZS(3)=0.5*(YGDOT(JK)+YGDOT(KK))
      ZS(4)=0.5*(BETA(JK)+BETA(KK))
      ZS(5)=0.5*(ACCXG(JK)+ACCXG(KK))
      ZS(6)=0.5*(ACCYG(JK)+ACCYG(KK))
      CALL  FUNC(X1,ZY,DZX,K,XCFF)
      DO  825  I=1,3
      DZY2(I)=DZX(I)
825   ZY(I)=ZX(I)+0.5*T*DZX(I)
      XCFF=0.5*(XCF(JK)+XCF(KK))
      CALL  FUNC(X1,ZY,DZX,K,XCFF)
      DO  830  I=1,3
      DZY3(I)=DZX(I)
830   ZY(I)=ZX(I)+T*DZX(I)
      XCFF=XCF(KK)
      XCFP=(XCDEL(KKJ)-XCDEL(KK))/T
      ZS(1)=PHYDOT(KK)
      ZS(2)=XGDOT(KK)
      ZS(3)=YGDOT(KK)
      ZS(4)=BETA(KK)
      ZS(5)=ACCXG(KK)
      ZS(6)=ACCYG(KK)
      CALL  FUNC(X2,ZY,DZX,K,XCFF)
      DO  835  I=1,3
      DZY4(I)=DZX(I)
      ZX(I)=ZX(I)+T*(DZY1(I)+2. *DZY2(I)+2. *DZY3(I)+DZY4(I))/6.0
835   ZZ(KK,I)=ZX(I)
      X=X+T
810   CONTINUE
      MK=3
      NK=3
      GO  TO  834
      END
```

```
*IBFTC FUNC
      SUBROUTINE  FUNC(X,Z,DZ,K,XCFF)
C     ******************************
C     THIS SUBROUTINE EVALUATES THE FUNCTION VALUE REQUIRED WHILE
C         APPLYING RUNGE-KUTTA METHOD FOR THE SOLUTION OF ANALYSIS [illegible].
      COMMON/TAJ/ZS
      COMMON/SMITA/XCFP
      COMMON/KHARE/R,AL,A,OI,GI,OM1,OM2,S,WN,A1,A2
      DIMENSION  ZS(6)
      DIMENSION  Z(3),DZ(3)
      XX=2.0*X
      P1=(A+R*SIN(X))/AL
      P2=A/AL
      P3=R/AL
      P4=P1**2
      P5=1.0-P4
      P6=SIN(X)**2
      P7=COS(X)**2
      P8=COS(X)**3
      P9=OI+GI*P3**2*P7/P5
      P10=OM2*R**2*(P6+0.25*P7*P4+0.5*SIN(XX)*P1)
      SS1=P9+P10
      P11=GI*P3**2*(2.0*P3*P8*P1-SIN(XX)*P5)/P5**2
      P12=OM2*R**2*(SIN(XX)+0.5*P3*P8*P1-0.25*SIN(XX)*P4)
      P13=OM2*R**2*(COS(XX)*P1+0.5*P3*SIN(XX)*COS(X))
      SS2=P11+P12+P13
      SS3=-R*COS(X)-AL*P5**0.5
      SS4=R*SIN(X)+R*COS(X)*P1
      SS5=WN**2*(R*COS(X)+AL*P5**0.5)
      H50=OM2*R**2/4.0
      H51=H50*COS(X)**2
      H52=H50*SIN(XX)
      SS1=SS1+H51
      SS2=SS2-H52
      DZ(1)=Z(3)
      P14=A1-S*(Z(1)+SS3)*(Z(3)+SS4)-Z(2)*(A2+0.5*SS2)
      DZ(2)=2.0*(P14-OM1*Z(3)*(SS5-Z(1)*WN**2))/SS1
      DZ(3)=(SS5-Z(1)*WN**2-0.5*Z(3)*DZ(2))/Z(2)
      IF(K.EQ.1)RETURN
      SS4=SS4-XCFP
      SS3=-XCFF
      SS5=WN**2*XCFF
      P14=A1-S*(Z(1)+SS3)*(Z(3)+SS4)-Z(2)*A2
      P15=GI*ZS(1)*ZS(4)+OM2*(ZS(2)*ZS(5)+ZS(3)*ZS(6))
      P16=-P15/SQRT(Z(2))
      P17=OM1*Z(3)*(Z(1)*WN**2-SS5)
      DZ(2)=2.0*(P14+P16+P17)/OI
      DZ(3)=(SS5-Z(1)*WN**2-0.5*Z(3)*DZ(2))/Z(2)
      RETURN
      END
```

*ENTRY

****
* INPUT DATA *
****

12.55	445.0	-[illegible]	765.[illegible]	[illegible].63	9.38	[illegible]	[illegible].6
1.0	0.153	0.0104	[illegible].052	2.5	64.5	1.[illegible]	.75
0.083	0.0469	0.0	21.0	0.01	5.0	0.005	0.000[illegible]
32.78	700.0	-0.56	1200.0	[illegible].922	36.961	2.59	1.[illegible]
24.0	5.1	0.052	[illegible].265	[illegible]	[illegible]	2.[illegible]	1.5
0.3334	0.188	0.0	25.0	.5	5.[illegible]	[illegible].01	.1
24.9	700.0	-0.5	1200.0	[illegible].829	19.164	0.688	1.[illegible]
24.0	5.1	0.052	0.0265	120.0	[illegible]	2.[illegible]	1.5
0.3334	0.188	[illegible].0	25.[illegible]	.5	5.[illegible]	0.01	.1

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